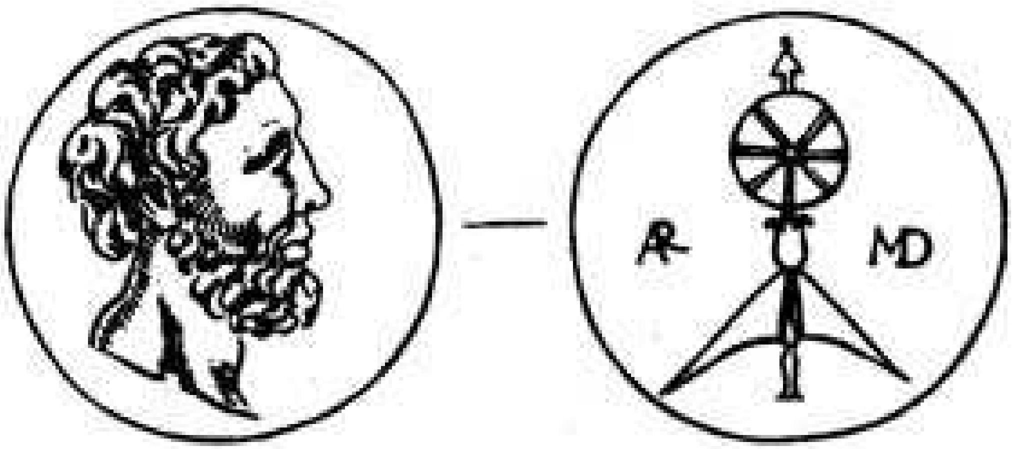
Archimedes …

Archimedes ...

The oldest portrait of Archimedes ...

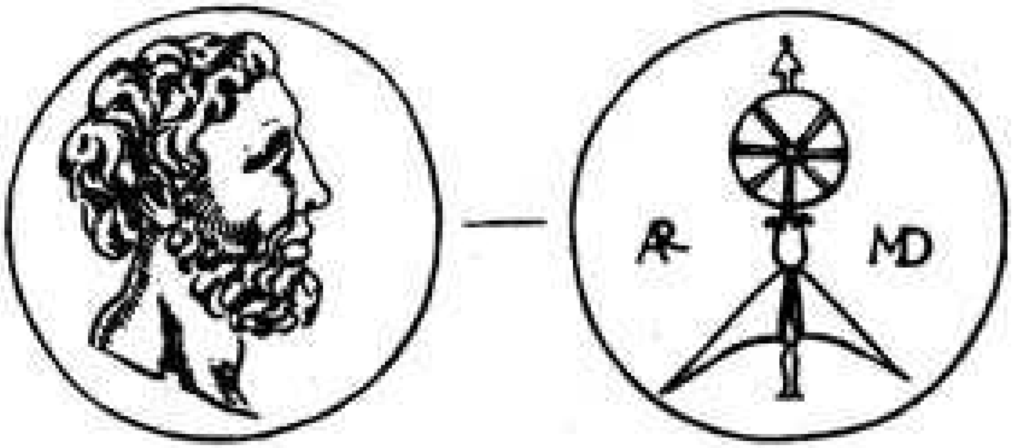


(Paruta, La Sicilia descritta con medaglie, 1612)

Archimedes ...

... and the newest

The oldest portrait of Archimedes ...

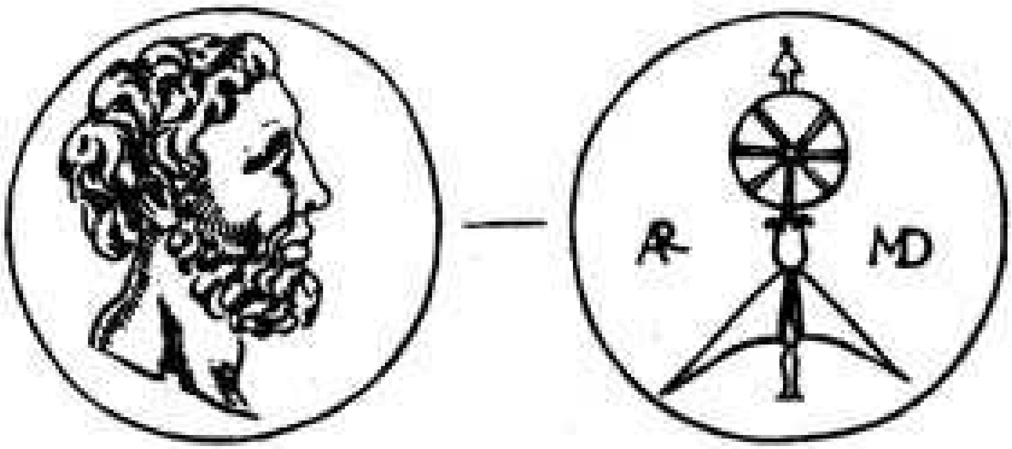


(Paruta, La Sicilia descritta con medaglie, 1612)

(Evi Hairer's reproduction
of Martin's Fields Medal)

Archimedes ...

The oldest portrait of Archimedes ...



(Paruta, La Sicilia descritta con medaglie, 1612)

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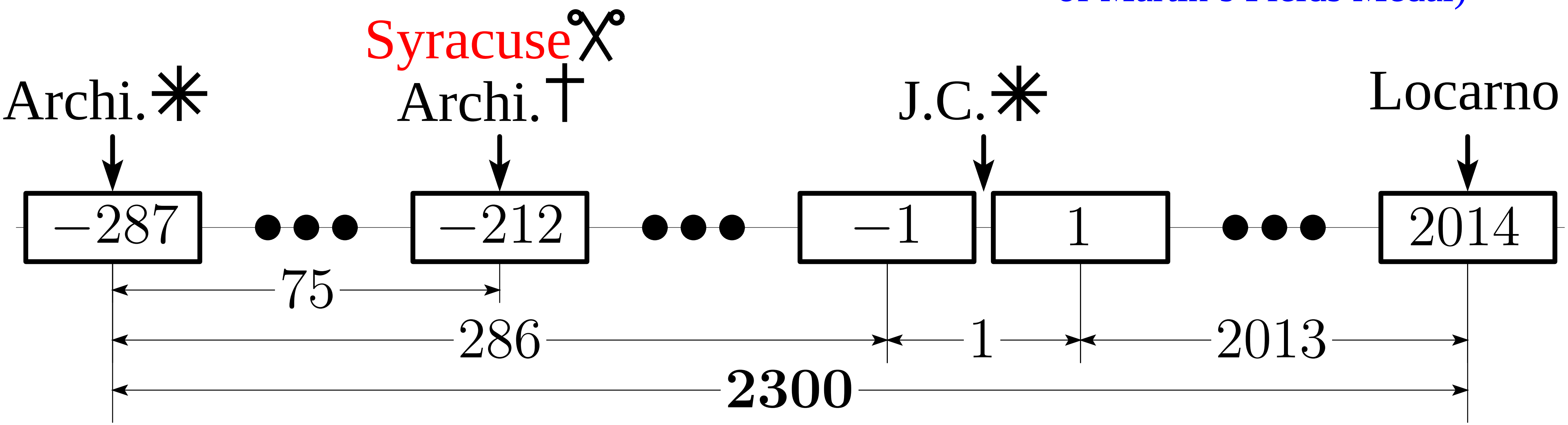
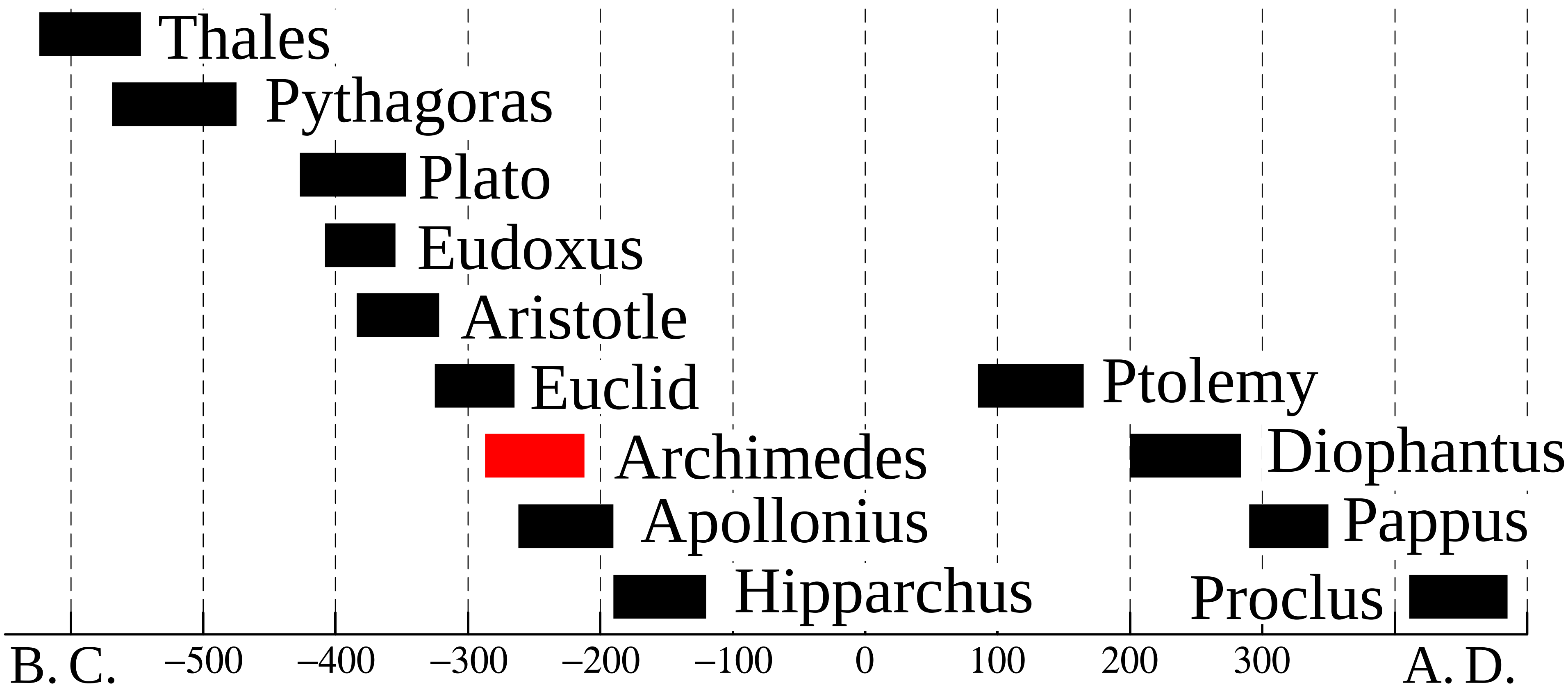
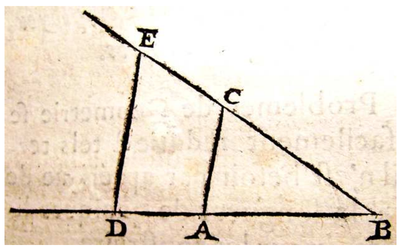
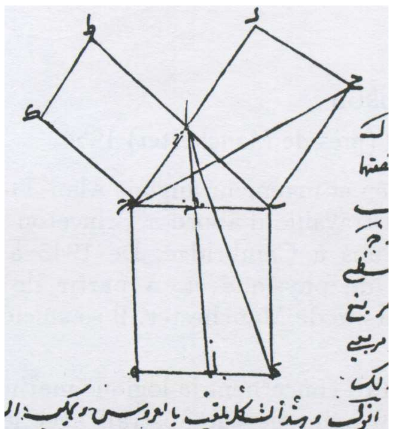


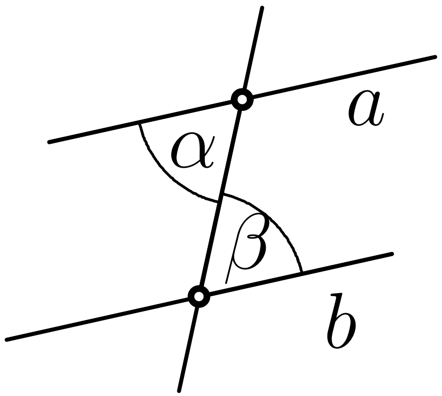
JEUNE de 2300 ans !

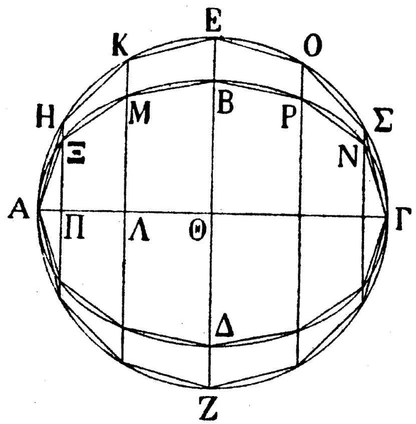
The ”Big Bang of Science” in Greece:

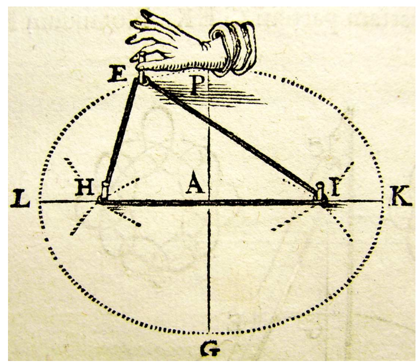










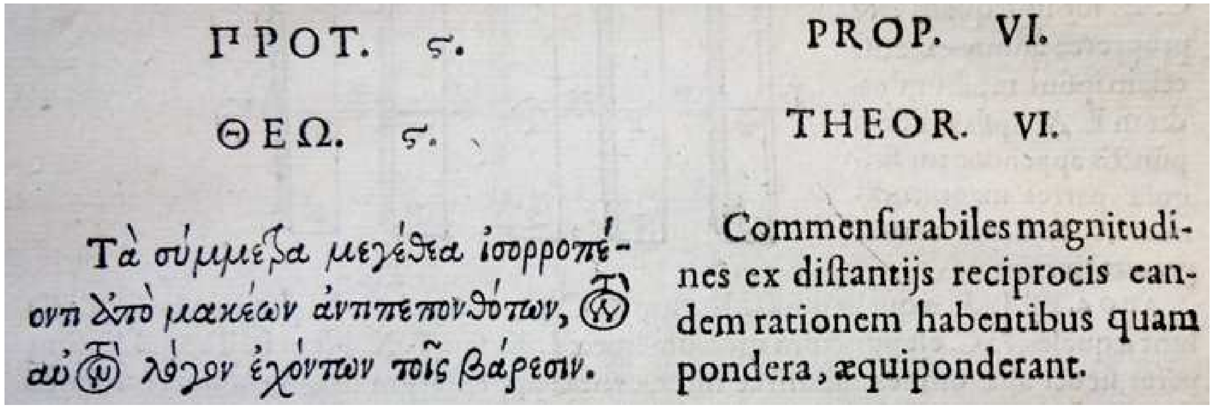


Equilibrium of planes.

ΑΡΧΙΜΗΔΟΥΣ

ΕΠΙΠΕΔΩΝ ΙΣΟΡΡΟ-
ΠΙΩΝ, Η ΚΕΝΤΡΑΒΑΡΩΝ
ΕΠΙΠΕΔΩΝ,

(Opera omnia, printed 1615 (Paris, ed. David Rivault, BGE Ka459)



ΠΡΟΤ. ς.

ΘΕΩ. ς.

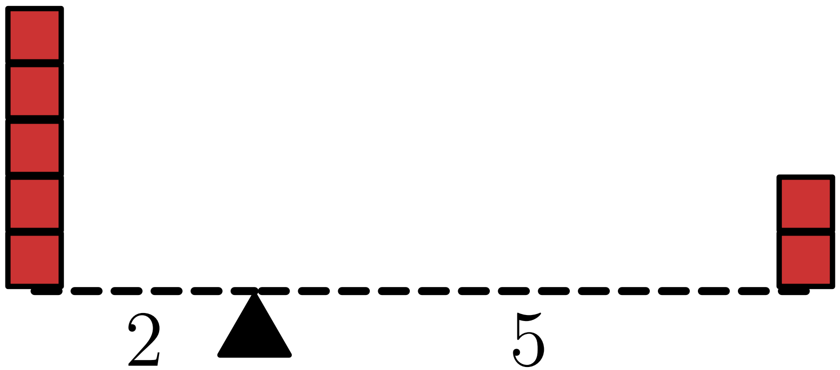
Τὰ σύμμετρα μεγέθεα ἰσορροπέ-
οντι ἀπὸ μακέων ἀντιπεπονθότων, τὸν
αὐτὸν λόγον ἐχόντων τοῖς βάρεσιν.

PROP. VI.

THEOR. VI.

Commenſurabiles magnitudi-
nes ex diſtantijs reciprocis ean-
dem rationem habentibus quam
pondera, æquiponderant.

(Opera omnia, printed 1615 (Paris, ed. David Rivault, BGE Ka459)



“Donne-moi où je puisse tenir ferme, et j’ébranlerai la terre” (trad. Ver Eecke)

Archimedes' original proof: (from edition Heiberg 1913)

ἔστω σύμμετρα μεγέθεα τὰ Α, Β, ὧν κέντρα τὰ Α, Β, καὶ μᾶκος ἔστω τι τὸ ΕΔ, καὶ ἔστω, ὡς τὸ Α ποτὶ τὸ Β, οὕτως τὸ ΔΓ μᾶκος ποτὶ τὸ ΓΕ μᾶκος· δεικτέον, ὅτι τοῦ ἐξ ἀμφοτέρων τῶν Α, Β συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος τὸ Γ.

ἐπεὶ γάρ ἐστιν, ὡς τὸ Α ποτὶ τὸ Β, οὕτως τὸ ΔΓ ποτὶ τὸ ΓΕ, τὸ δὲ Α τῷ Β σύμμετρον, καὶ τὸ ΓΔ ἄρα τῷ ΓΕ σύμμετρον, τουτέστιν εὐθεῖα τᾷ εὐθείᾳ· ὥστε τῶν ΕΓ, ΓΔ ἐστι κοινὸν μέτρον. ἔστω δὴ τὸ Ν, καὶ κείσθω τᾷ μὲν ΕΓ ἴσα ἑκατέρα τᾶν ΔΗ, ΔΚ, τᾷ δὲ ΔΓ ἴσα ἁ ΕΛ. καὶ ἐπεὶ ἴσα ἁ ΔΗ τᾷ ΓΕ, ἴσα καὶ ἁ ΔΓ τᾷ ΕΗ· ὥστε καὶ ἁ ΛΕ ἴσα τᾷ ΕΗ. διπλασία ἄρα ἁ μὲν ΛΗ τᾶς ΔΓ, ἁ δὲ ΗΚ τᾶς ΓΕ· ὥστε τὸ Ν καὶ ἑκατέραν τᾶν ΛΗ, ΗΚ μετρεῖ, ἐπειδήπερ καὶ τὰ ἡμίσεα αὐτᾶν. καὶ ἐπεί ἐστιν, ὡς τὸ Α ποτὶ τὸ Β, οὕτως ἁ ΔΓ ποτὶ ΓΕ, ὡς δὲ ἁ ΔΓ ποτὶ ΓΕ, οὕτως ἁ ΛΗ ποτὶ ΗΚ· διπλασία γὰρ ἑκατέρα ἑκατέρας· καὶ ὡς ἄρα τὸ Α ποτὶ τὸ Β, οὕτως ἁ ΛΗ ποτὶ ΗΚ. ὁσαπλασίων δέ ἐστιν ἁ ΛΗ τᾶς Ν, τοσαυταπλασίων ἔστω καὶ τὸ Α τοῦ Ζ· ἔστιν ἄρα, ὡς ἁ ΛΗ ποτὶ Ν, οὕτως τὸ Α ποτὶ Ζ. ἔστι δὲ καί, ὡς ἁ ΚΗ ποτὶ ΛΗ, οὕτως τὸ Β ποτὶ Α· δι' ἴσου ἄρα ἐστίν, ὡς ἁ ΚΗ ποτὶ Ν, οὕτως τὸ Β ποτὶ Ζ· ἰσάκις ἄρα πολλαπλασίων ἐστὶν ἁ ΚΗ τᾶς Ν καὶ τὸ Β τοῦ Ζ. ἐδείχθη δὲ τοῦ Ζ καὶ τὸ Α πολλαπλάσιον ἐόν· ὥστε τὸ Ζ τῶν Α, Β κοινόν ἐστι μέτρον. διαιρεθείσας οὖν τᾶς μὲν ΛΗ εἰς τὰς τᾷ Ν ἴσας, τοῦ δὲ Α εἰς τὰ τῷ Ζ ἴσα, τὰ ἐν τᾷ ΛΗ τμάματα ἰσομεγέθεα τᾷ Ν ἴσα ἐσσεῖται τῷ πλήθει τοῖς ἐν τῷ Α τμαμάτεσσιν ἴσοις ἐοῦσιν τῷ Ζ. ὥστε, ἂν ἐφ' ἕκαστον τῶν τμαμάτων τῶν ἐν τᾷ ΛΗ ἐπιτεθῇ μέγεθος ἴσον τῷ Ζ τὸ κέντρον τοῦ βάρεος ἔχον ἐπὶ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐντὶ τῷ Α, καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον ἐσσεῖται τοῦ βάρεος τὸ Ε· ἄρτιά τε γάρ ἐστι τὰ πάντα τῷ πλήθει, καὶ τὰ ἐφ' ἑκάτερα τοῦ Ε ἴσα τῷ πλήθει διὰ τὸ ἴσαν εἶμεν τὰν ΛΕ τᾷ ΗΕ. ὁμοίως δὲ δειχθήσεται, ὅτι κἄν, εἴ κα ἐφ' ἕκαστον τῶν ἐν τᾷ ΚΗ τμαμάτων ἐπιτεθῇ μέγεθος ἴσον τῷ Ζ κέντρον τοῦ βάρεος ἔχον ἐπὶ τοῦ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐσσεῖται τῷ Β, καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον τοῦ βάρεος ἐσσεῖται τὸ Δ· ἐσσεῖται οὖν τὸ μὲν Α ἐπικείμενον κατὰ τὸ Ε, τὸ δὲ Β κατὰ τὸ Δ. ἐσσεῖται δὴ μεγέθεα ἴσα ἀλλάλοις ἐπ' εὐθείας κείμενα, ὧν τὰ κέντρα τοῦ βάρεος ἴσα ἀπ' ἀλλάλων διέστακεν, [συγκείμενα] ἄρτια τῷ πλήθει· δῆλον οὖν, ὅτι τοῦ ἐκ πάντων συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος ἁ διχοτομία τᾶς εὐθείας τᾶς ἐχούσας τὰ κέντρα τῶν μέσων μεγεθέων. ἐπεὶ δ' ἴσαι ἐντὶ ἁ μὲν ΛΕ τᾷ ΓΔ, ἁ δὲ ΕΓ τᾷ ΔΚ, καὶ ὅλα ἄρα ἁ ΛΓ ἴσα τᾷ ΓΚ· ὥστε τοῦ ἐκ πάντων μεγέθεος κέντρον τοῦ βάρεος τὸ Γ σαμεῖον. τοῦ μὲν ἄρα Α κειμένου κατὰ τὸ Ε, τοῦ δὲ Β κατὰ τὸ Δ, ἰσορροπησοῦντι κατὰ τὸ Γ.

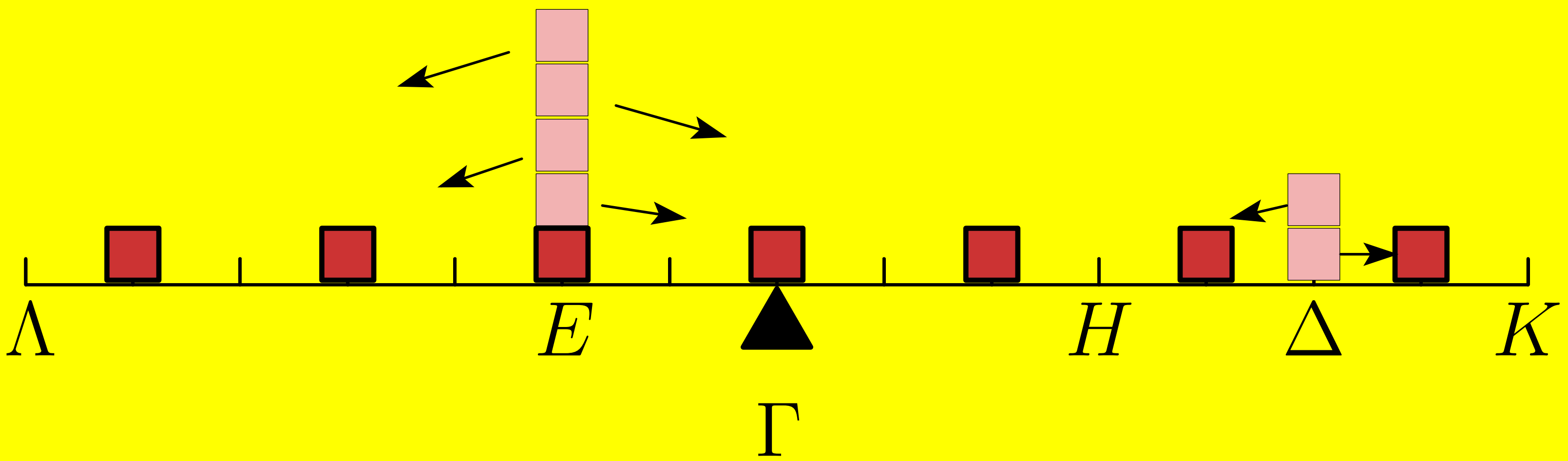
Archimedes' original proof: (from edition Heiberg 1913)

ἔστω σύμμετρα μεγέθεα τὰ Α, Β, ὧν κέντρα τὰ Α, Β, καὶ μᾶκος ἔστω τι τὸ ΕΔ, καὶ ἔστω, ὡς τὸ Α ποτὶ τὸ Β, οὕτως τὸ ΔΓ μᾶκος ποτὶ τὸ ΓΕ μᾶκος· δεικτέον, ὅτι τοῦ ἐξ ἀμφοτέρων τῶν Α, Β συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος τὸ Γ.

ἐπεὶ γάρ ἐστιν, ὡς τὸ Α ποτὶ τὸ Β, οὕτως τὸ ΔΓ ποτὶ τὸ ΓΕ, τὸ δὲ Α τῷ Β σύμμετρον, καὶ τὸ ΓΔ
ἄρα τῷ ΓΕ …
ὥστε τῶν Γ…
Ν, καὶ κείσ…
τᾷ δὲ ΔΓ ἴ…
καὶ ἁ ΔΓ τ…
πλασία ἄρα …
ὥστε τὸ Ν …
δήπερ καὶ τ…
ποτὶ τὸ Β, …
ΓΕ, οὕτως …
ἑκατέρας· κ…
ποτὶ ΗΚ. δ…
ταπλασίων ἔστω καὶ τὸ Α τοῦ Ζ· ἔστιν ἄρα, ὡς ἁ ΛΗ ποτὶ Ν, οὕτως τὸ Α ποτὶ Ζ. ἔστι δὲ καί, ὡς ἁ ΚΗ ποτὶ ΛΗ, οὕτως τὸ Β ποτὶ Α· δι' ἴσου ἄρα ἐστίν, ὡς ἁ ΚΗ ποτὶ Ν, οὕτως τὸ Β ποτὶ Ζ· ἰσάκις ἄρα πολλαπλασίων ἐστὶν ἁ ΚΗ τᾶς Ν καὶ τὸ Β τοῦ Ζ. ἐδείχθη δὲ τοῦ Ζ καὶ τὸ Α πολλαπλάσιον ἐόν· ὥστε τὸ Ζ τῶν Α, Β κοινόν ἐστι μέτρον. διαιρεθείσας οὖν τᾶς μὲν ΛΗ εἰς τὰς τᾷ Ν ἴσας, τοῦ δὲ Α

εἰς τὰ τῷ Ζ ἴσα, τὰ ἐν τᾷ ΛΗ τμάματα ἰσομεγέθεα τᾷ Ν ἴσα ἐσσεῖται τῷ πλήθει τοῖς ἐν τῷ Α τμαμάτεσσιν ἴσοις ἐοῦσιν τῷ Ζ. ὥστε, ἂν ἐφ' ἕκαστον τῶν τμαμάτων τῶν ἐν τᾷ ΛΗ ἐπιτεθῇ μέγεθος ἴσον τῷ Ζ τὸ κέντρον τοῦ βάρεος ἔχον ἐπὶ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐντὶ τῷ Α, καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον ἐσσεῖται τοῦ βάρεος τὸ Ε·
…ὶ τὰ ἐφ'
…ίμεν τὰν
…ν, εἴ κα
…θῇ μέγε-
…ὶ τοῦ μέ-
…ἐσσεῖται
…τρον τοῦ
…ν Α ἐπι-
…ἐσσεῖται
…κ, ὧν τὰ
…εν, [συγ-
… τοῦ ἐκ πάντων συγκειμένου μεγέθεος κέντρον ἐστὶ τοῦ βάρεος ἁ διχοτομία τᾶς εὐθείας τᾶς ἐχούσας τὰ κέντρα τῶν μέσων μεγεθέων. ἐπεὶ δ' ἴσαι ἐντὶ ἁ μὲν ΛΕ τᾷ ΓΔ, ἁ δὲ ΕΓ τᾷ ΔΚ, καὶ ὅλα ἄρα ἁ ΛΓ ἴσα τᾷ ΓΚ· ὥστε τοῦ ἐκ πάντων μεγέθεος κέντρον τοῦ βάρεος τὸ Γ σαμεῖον. τοῦ μὲν ἄρα Α κειμένου κατὰ τὸ Ε, τοῦ δὲ Β κατὰ τὸ Δ, ἰσορροπησοῦντι κατὰ τὸ Γ.

Or a bit clearer as a picture ...



Archimedes' original proof:

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ἐπεὶ γάρ ἐστιν, ὡς τὸ Α ποτὶ τὸ Β, οὕτως τὸ ΔΓ ποτὶ τὸ ΓΕ, τὸ δὲ Α τῷ Β σύμμετρον, καὶ τὸ ΓΔ

ἄρα τῷ ΓΕ

ὥστε τῶν Γ

Ν, καὶ κείσ

τᾷ δὲ ΔΓ ἴ

καὶ ἁ ΔΓ τ

πλασία ἄρα

ὥστε τὸ Ν

δήπερ καὶ τ

ποτὶ τὸ Β,

ΓΕ, οὕτως

ἑκατέρας· κ

ποτὶ ΗΚ. ὁ

ταπλασίων

ΔΗ ποτὶ Ν

ΚΗ ποτὶ

ἐστίν, ὡς ἁ ΚΗ ποτὶ Ν, οὕτως τὸ Β ποτὶ Ζ· ἰσάκις ἄρα πολλαπλασίων ἐστὶν ἁ ΚΗ τᾶς Ν καὶ τὸ Β τοῦ Ζ. ἐδείχθη δὲ τοῦ Ζ καὶ τὸ Α πολλαπλάσιον ἐόν· ὥστε τὸ Ζ τῶν Α, Β κοινόν ἐστι μέτρον. διαιρεθείσας οὖν τᾶς μὲν ΔΗ εἰς τὰς τᾷ Ν ἴσας, τοῦ δὲ Α εἰς τὰ τῷ Ζ ἴσα, τὰ ἐν τᾷ ΔΗ τμάματα ἰσομεγέθεα τᾷ Ν ἴσα ἐσσεῖται τῷ πλήθει τοῖς ἐν τῷ Α τμαμάτεσσιν ἴσοις ἐοῦσιν τῷ Ζ. ὥστε, ἂν ἐφ' ἕκαστον τῶν τμαμάτων τῶν ἐν τᾷ ΔΗ ἐπιτεθῇ μέγεθος ἴσον τῷ Ζ τὸ κέντρον τοῦ βάρεος ἔχον ἐπὶ μέσου τοῦ τμάματος, τά τε πάντα μεγέθεα ἴσα ἐντὶ τῷ Α, καὶ τοῦ ἐκ πάντων συγκειμένου κέντρον ἐσσεῖται τοῦ βάρεος τὸ Ε·

ὶ τὰ ἐφ'

ῖμεν τὰν

ν, εἴ κα

θῇ μέγε-

ὶ τοῦ μέ-

ἐσσεῖται

τρον τοῦ

ν Α ἐπι-

ἐσσεῖται

α, ὧν τὰ

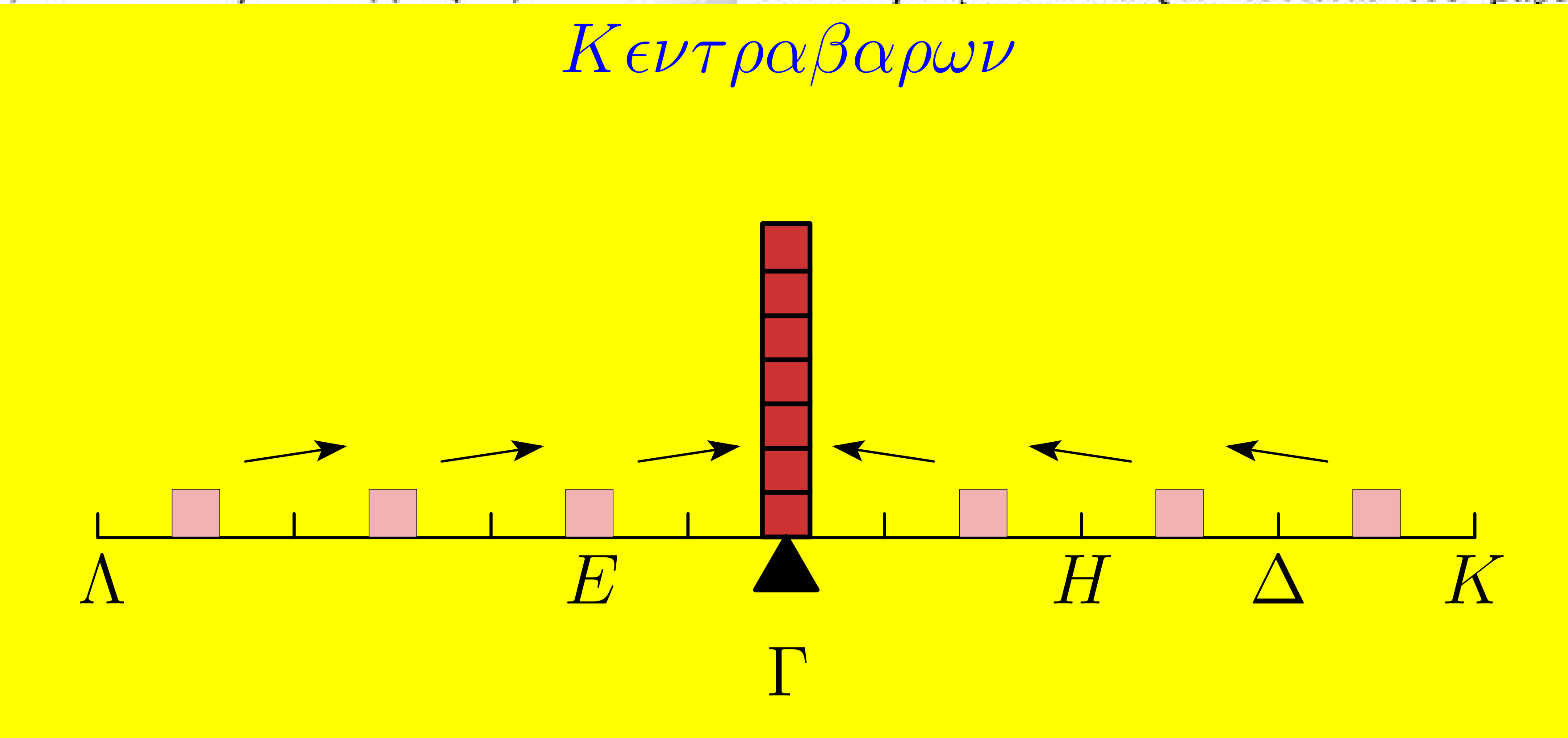
ιν, [συγ-

ὶ τοῦ ἐκ

τοῦ βά-

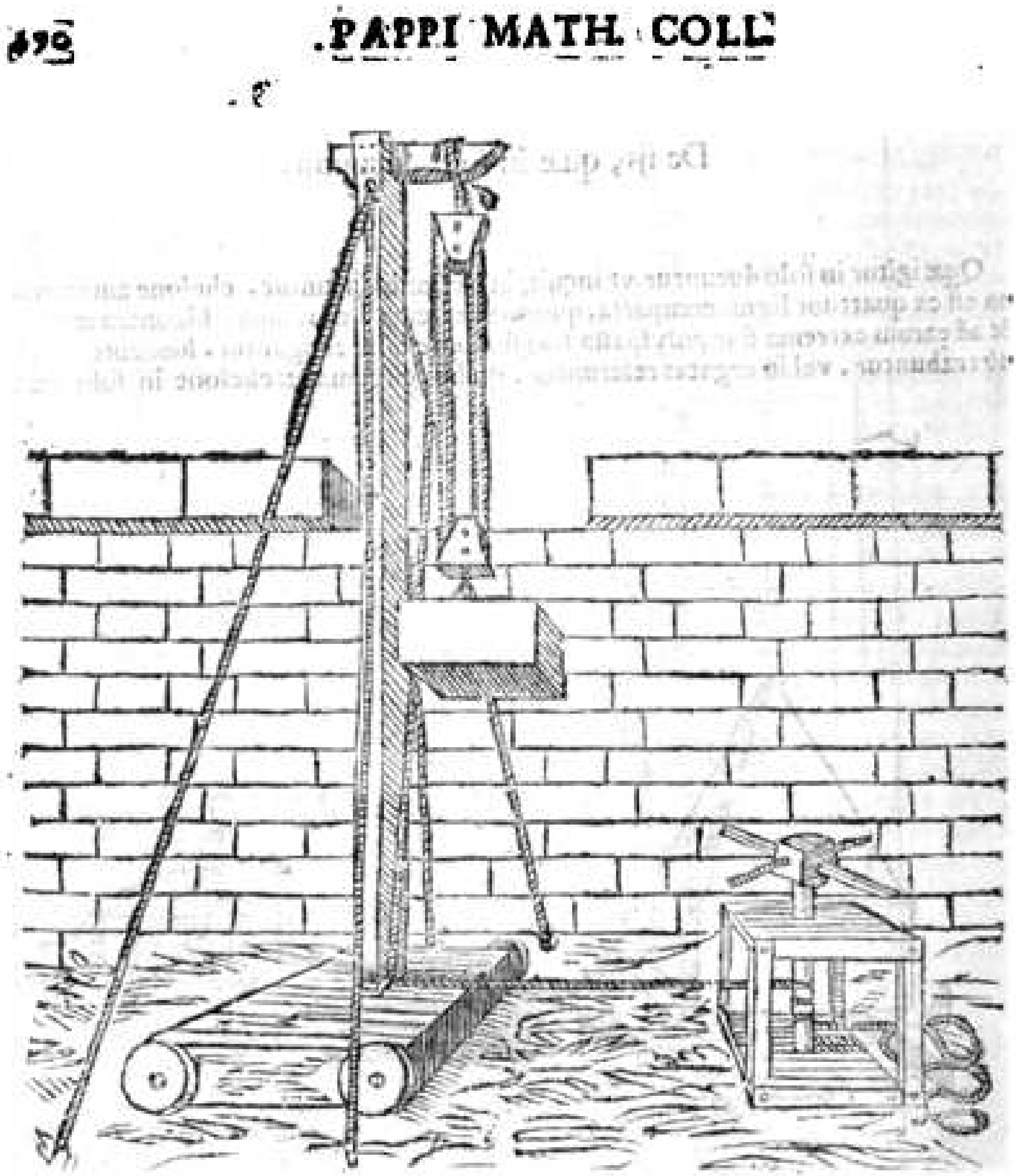
ὰ κέντρα

τῶν μέσων μεγεθέων. ἐπεὶ δ' ἴσαι ἐντὶ ἁ μὲν ΔΕ τᾷ ΓΔ, ἁ δὲ ΕΓ τᾷ ΔΚ, καὶ ὅλα ἄρα ἁ ΔΓ ἴσα τᾷ ΓΚ· ὥστε τοῦ ἐκ πάντων μεγέθεος κέντρον τοῦ βάρεος τὸ Γ σαμεῖον. τοῦ μὲν ἄρα Α κειμένου κατὰ τὸ Ε, τοῦ δὲ Β κατὰ τὸ Δ, ἰσορροπησοῦντι κατὰ τὸ Γ.

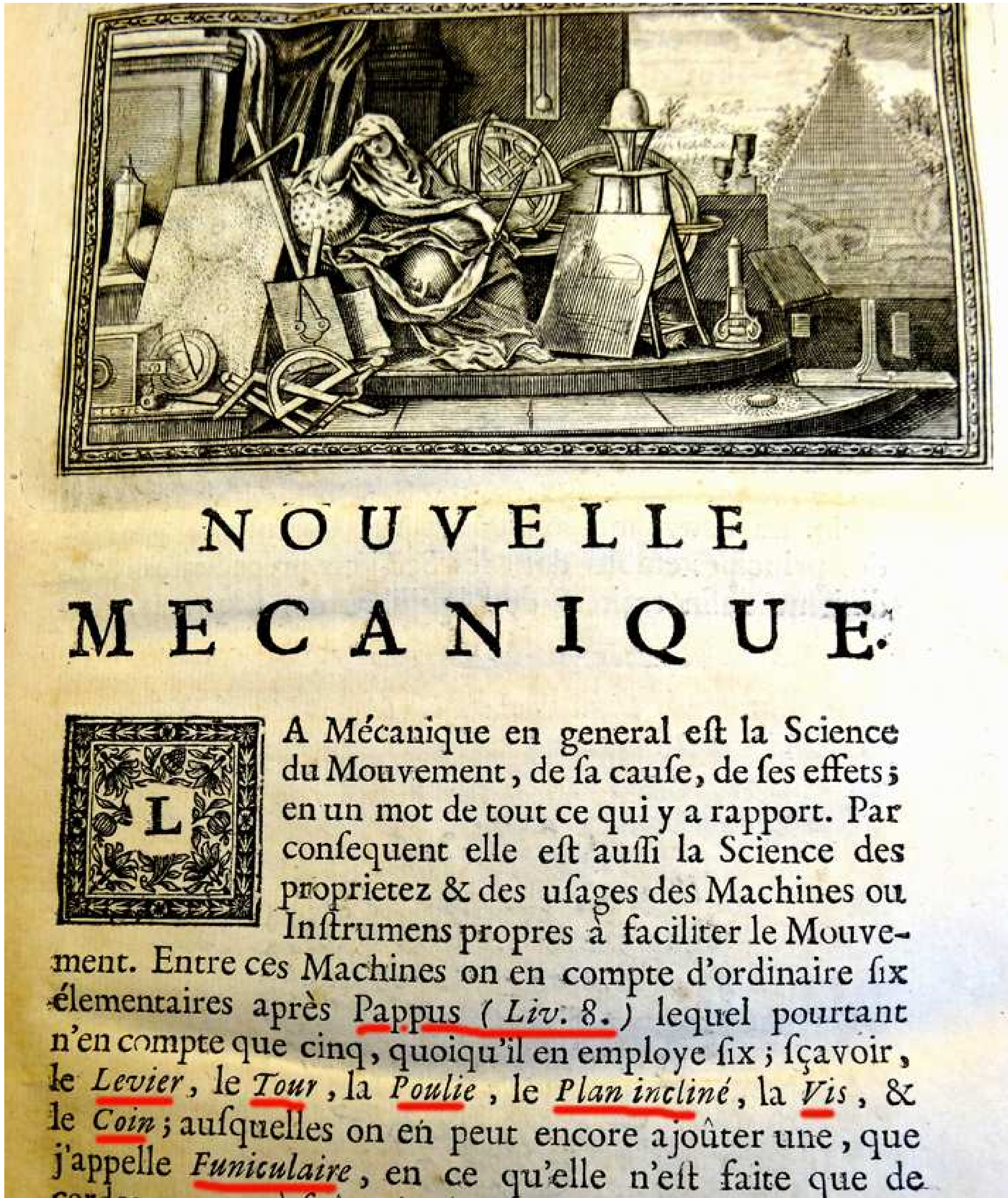


Pappus' *Collectiones* (≈ A.D. 300; Book VIII):

more and more complicated "machines" paved the road ...



Pappus' very last page



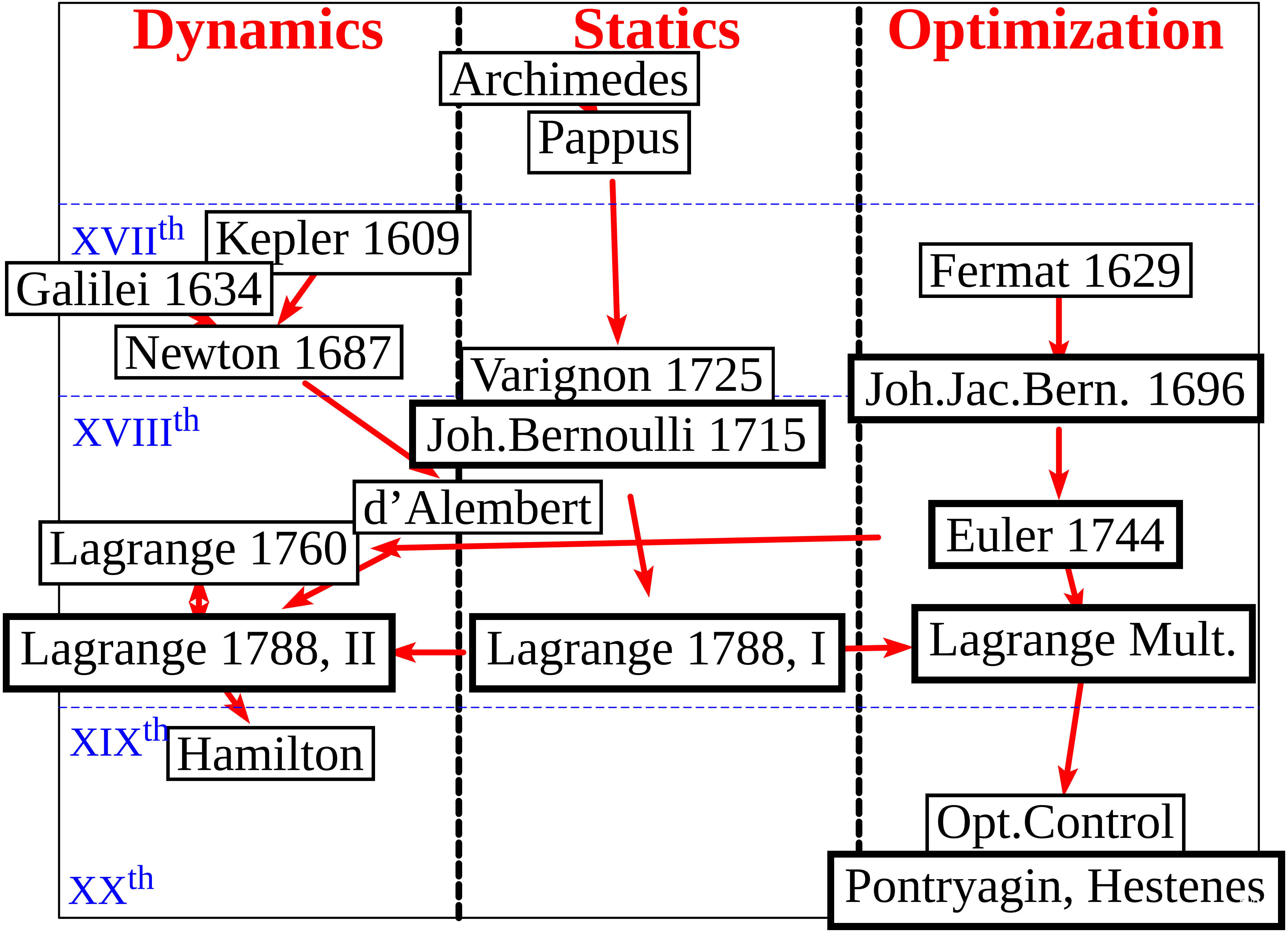
NOUVELLE

MECANIQUE.

LA Mécanique en general eſt la Science du Mouvement, de ſa cauſe, de ſes effets; en un mot de tout ce qui y a rapport. Par conſequent elle eſt auſſi la Science des proprietez & des uſages des Machines ou Inſtrumens propres à faciliter le Mouvement. Entre ces Machines on en compte d'ordinaire ſix élementaires après Pappus (*Liv. 8.*) lequel pourtant n'en compte que cinq, quoiqu'il en employe ſix; ſçavoir, le *Levier*, le *Tour*, la *Poulie*, le *Plan incliné*, la *Vis*, & le *Coin*; auſquelles on en peut encore ajoûter une, que j'appelle *Funiculaire*, en ce qu'elle n'eſt faite que de

First page of Varignon 1725

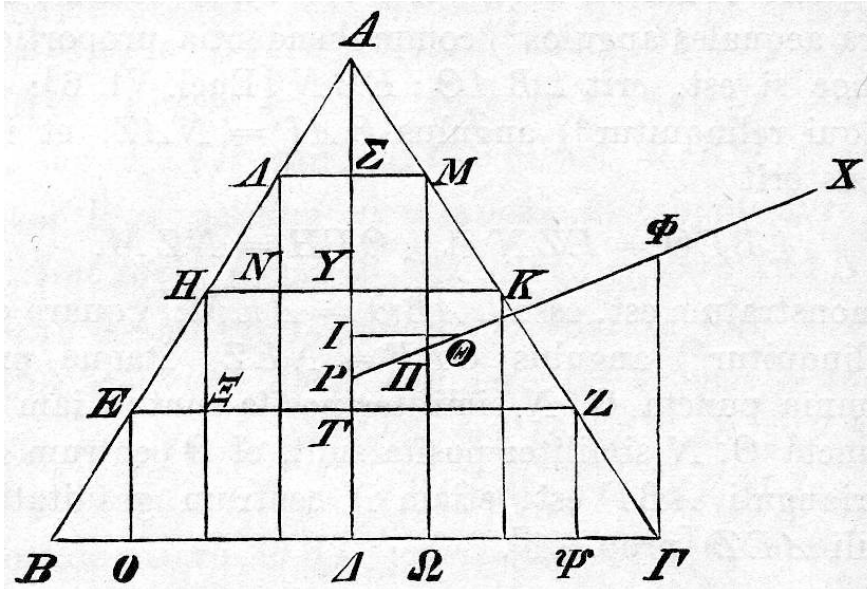
... to the origin of today's physics:

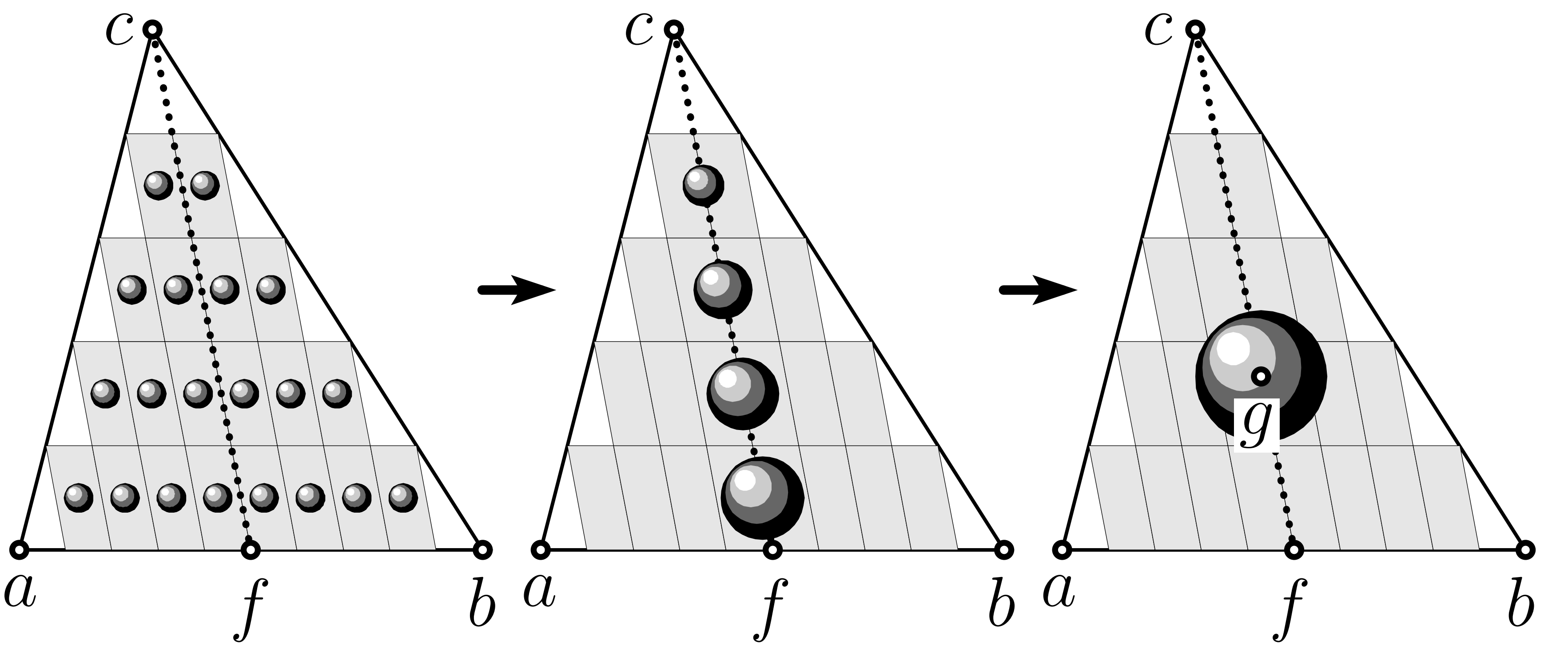


The barycentre of a plane triangle (Prop. 13):

The barycentre
of a plane triangle
lies on all medians;

⇒ All medians meet in G.
G divides medians in ratio $2 : 1$



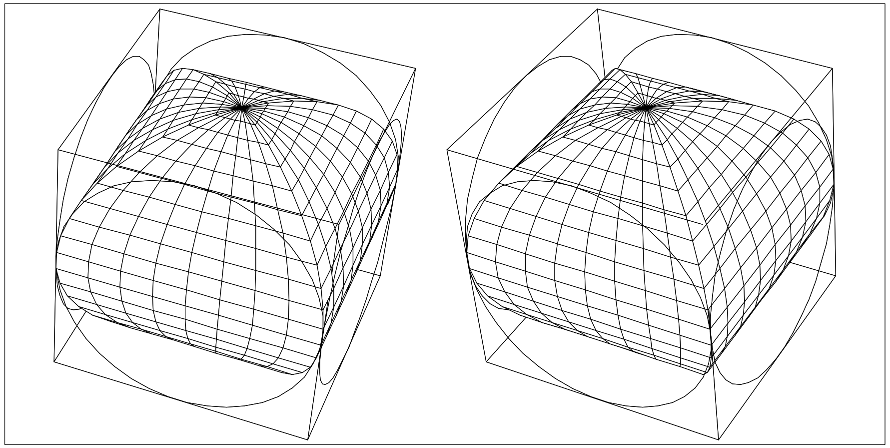


“Finite Element Method” and convergence proof!

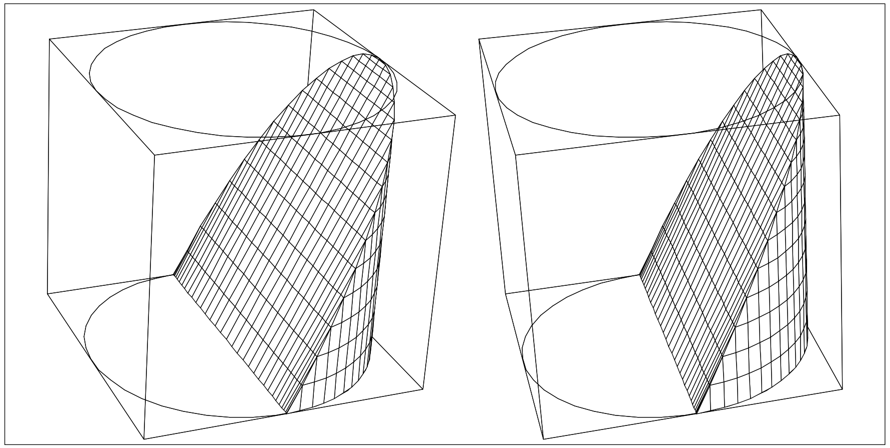
Archimedes' challenges for Eratosthenes:

"The method of mechanical theorems", sent to Eratosthenes ("Since I know that you are diligent ($\sigma\pi o\upsilon\delta\alpha\tilde{\iota}o\nu$), an excellent teacher of philosophy");

(discovered 1899 by Heiberg in a "Jerusalem palimpsest".)



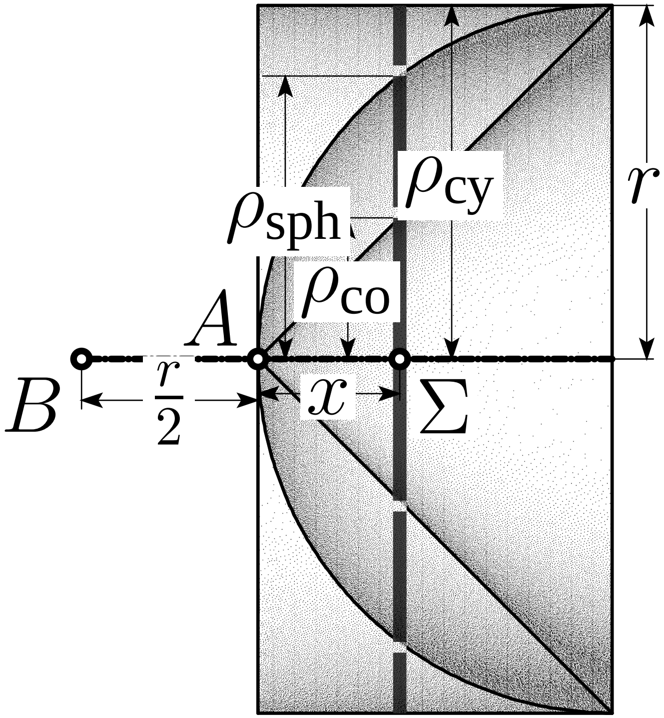
show that volume $= \dfrac{2}{3}$,

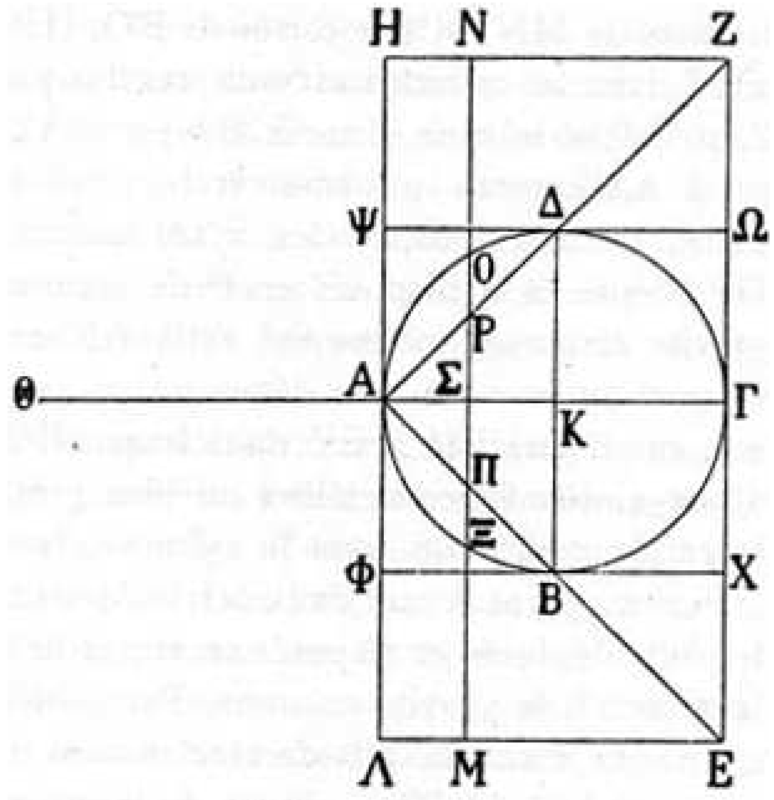


show that volume $= \dfrac{1}{6}$.

Archimedes' "weighting" of the sphere.

Idea: Place sphere, cone and cylinder "on a balance" at point A:



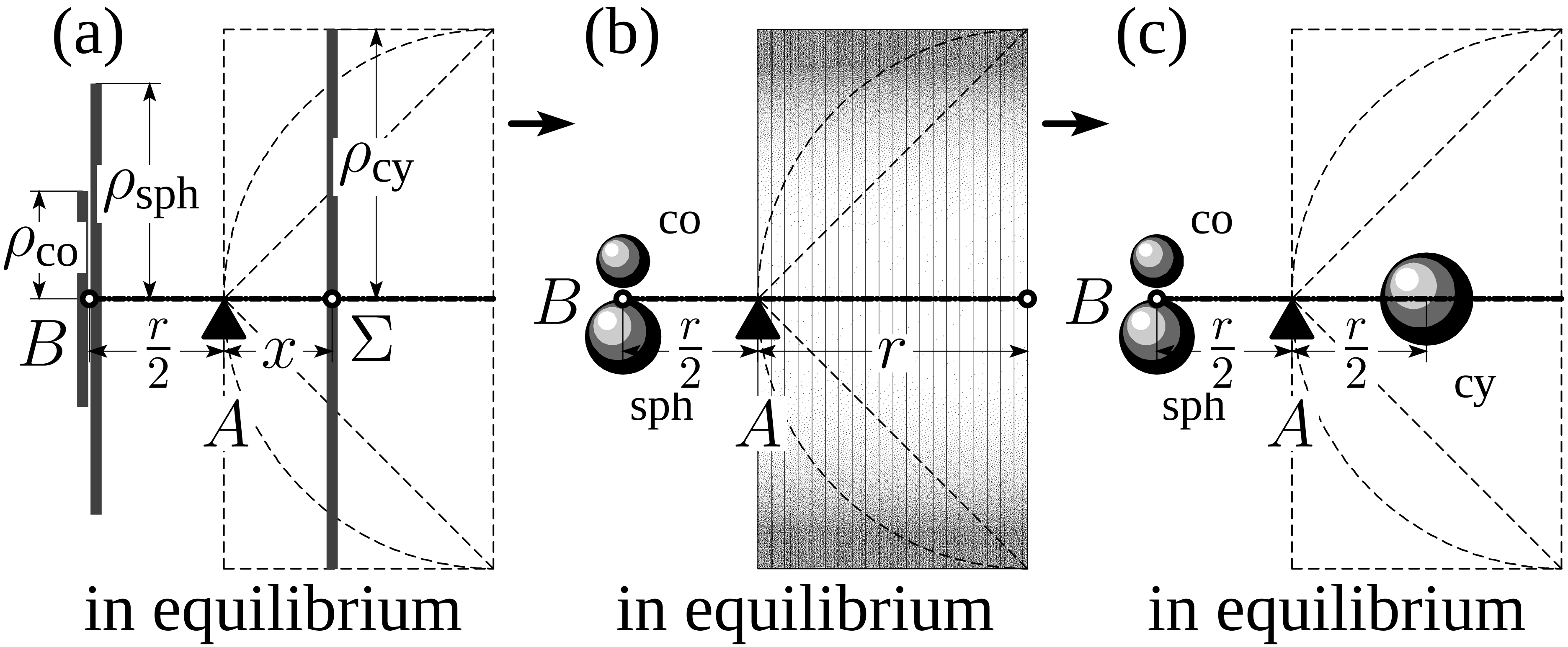


$$\text{Eucl. III.35: } \rho_{\text{sph}}^2 = x(2r - x) = 2\rho_{\text{cy}}^2 \frac{x}{r} - \rho_{\text{co}}^2$$

or

$$(\rho_{\text{sph}}^2 + \rho_{\text{co}}^2) \cdot \frac{r}{2} = \rho_{\text{cy}}^2 \cdot x \,.$$

$$(\rho_{\text{sph}}^2 + \rho_{\text{co}}^2) \cdot \frac{r}{2} = \rho_{\text{cy}}^2 \cdot x\,.$$



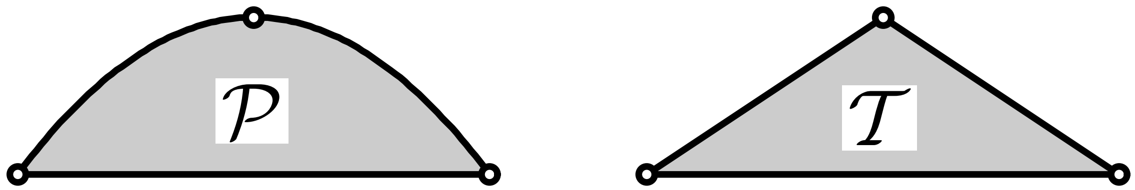
$\Rightarrow \mathcal{V}_{\text{sphere}} + \mathcal{V}_{\text{cone}} = \mathcal{V}_{\text{cylinder}}\,,$ (Eudoxus: $\mathcal{V}_{\text{cone}} = \frac{1}{3}\mathcal{V}_{\text{cylinder}}$)

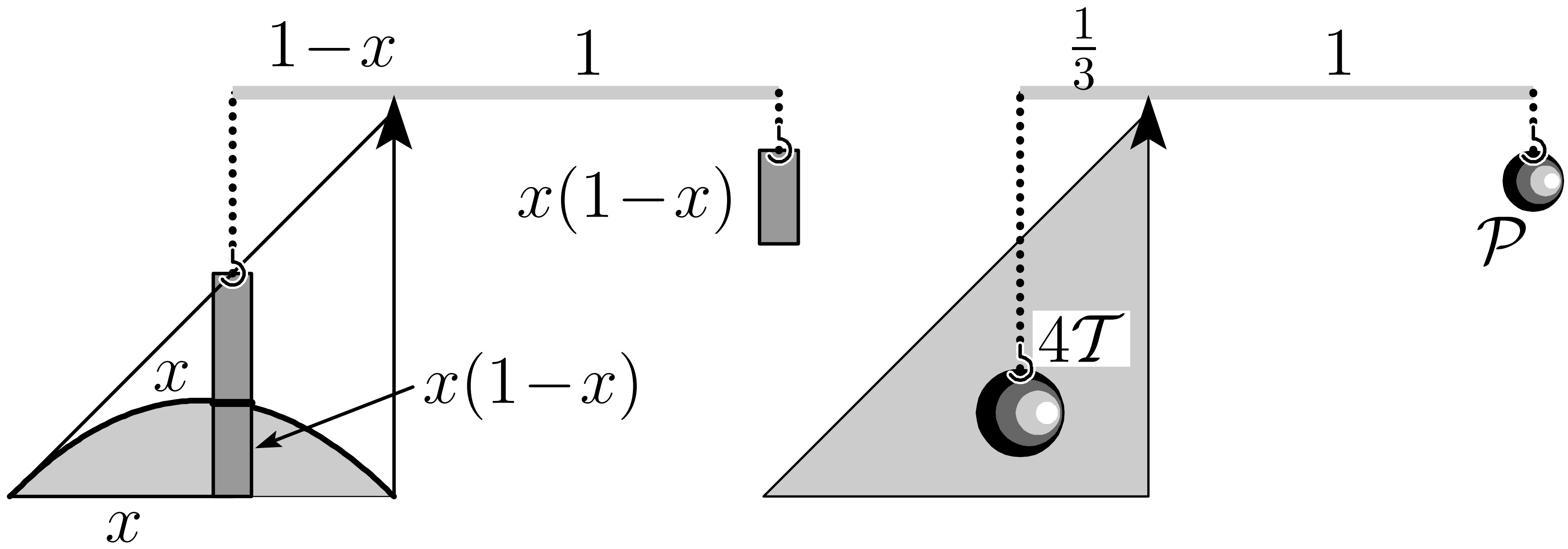
$$\Rightarrow \boxed{\mathcal{V}_{\text{cone}} : \mathcal{V}_{\text{sphere}} : \mathcal{V}_{\text{cylinder}} = 1 : 2 : 3}\,.$$

Cicéron: “Je trouvais (sa tombe) à la faveur de quelques vers ... et au-dessus une sphère et un cylindre” (Ver Eecke p. xxix)

Archimedes' "weighting" of the parabola.

which relation?



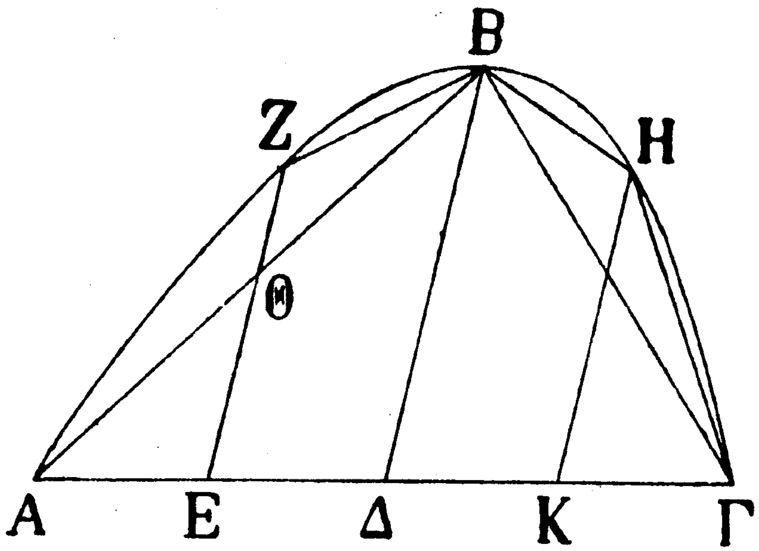


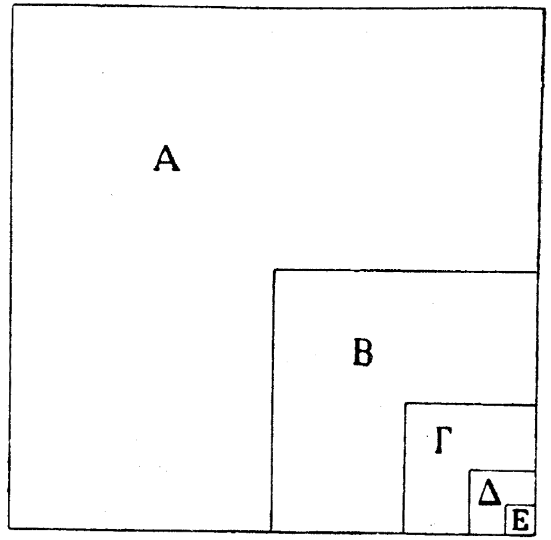
in equilibrium

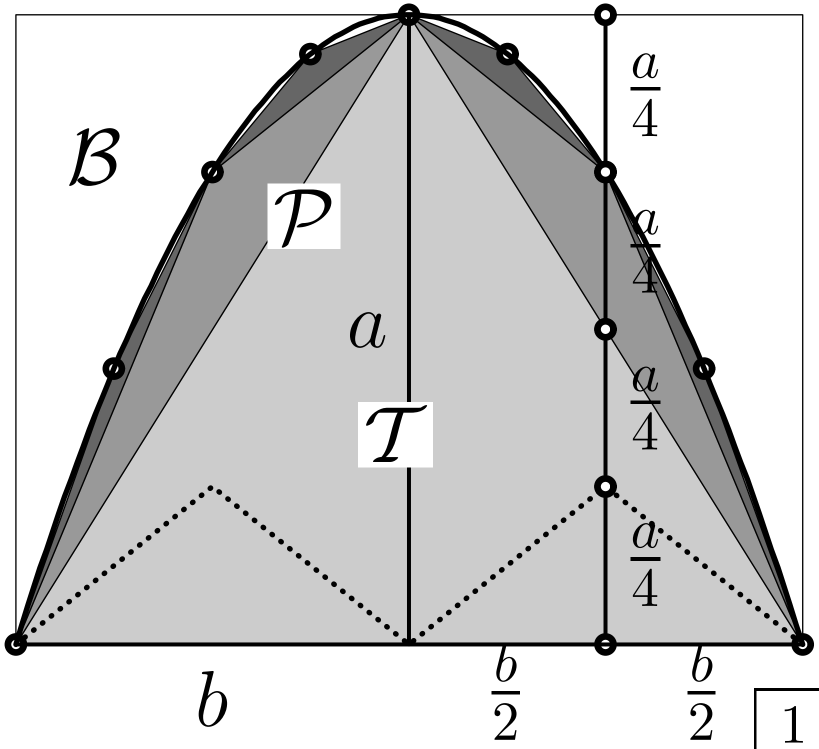
$$\Rightarrow \boxed{\mathcal{P} = \frac{4}{3}\mathcal{T}}.$$

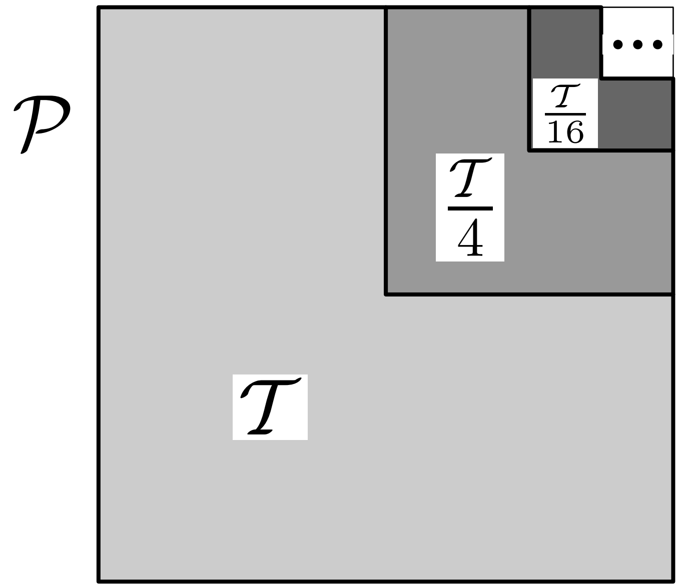
Arch.: "Is that rigorous?" **NO!** Rigorous is only Geometry $\Rightarrow$

Second proof: **The Quadrature of the Parabola.**









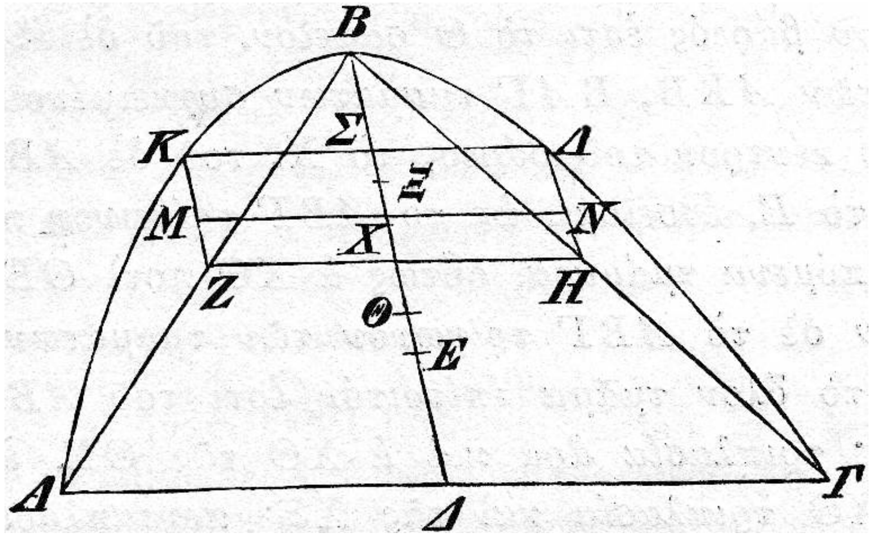
$$\frac{1}{4}\mathcal{P} = \frac{1}{3}\mathcal{T}$$

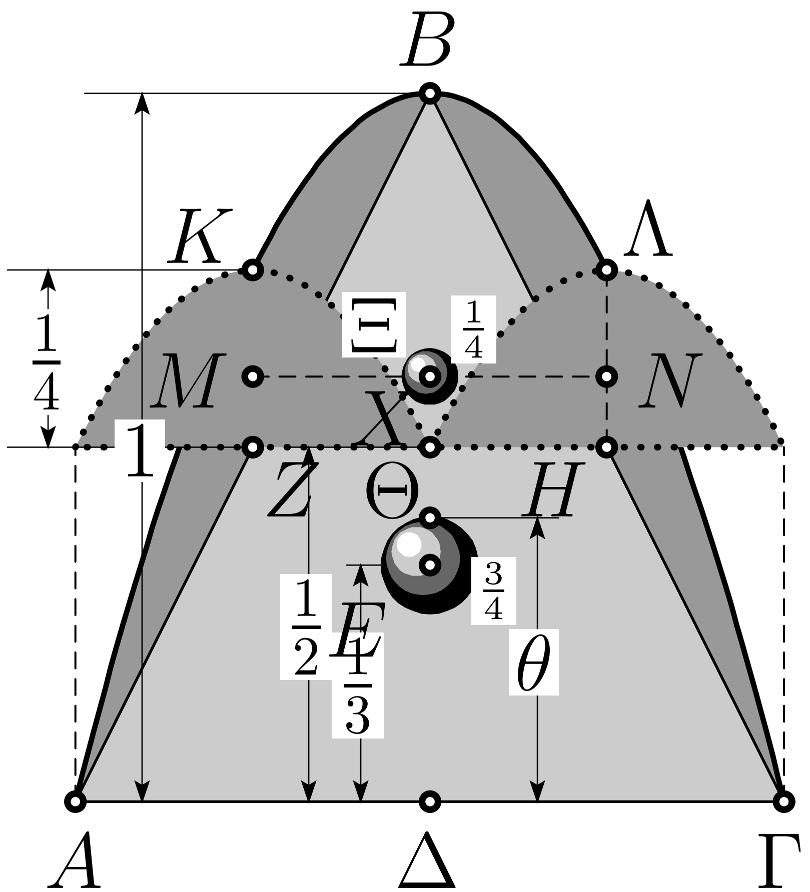
Equilibrium of planes II. Barycenter of parabola:

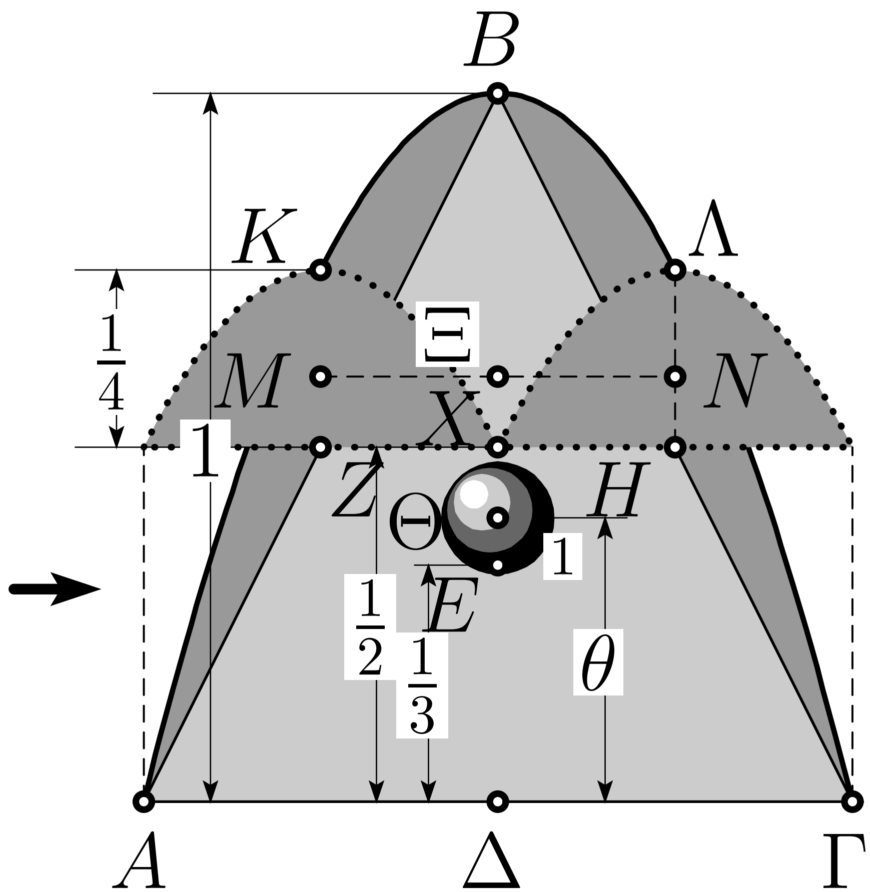
$$\Xi X = \tfrac{1}{4}\theta \quad \text{(Prop. 7)}$$

$$\theta = \frac{1}{4}\left(\frac{1}{2} + \frac{1}{4}\theta\right) + \frac{3}{4}\cdot\frac{1}{3}$$

$$\Rightarrow \boxed{\theta = \frac{2}{5}}$$

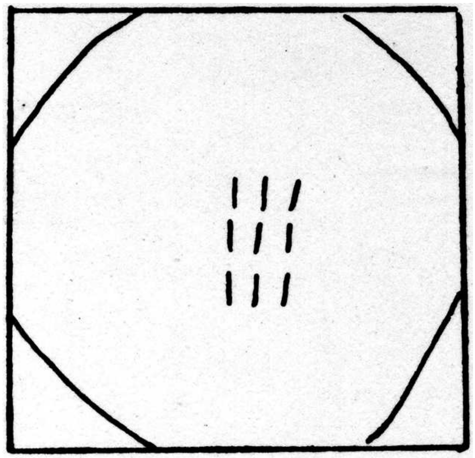


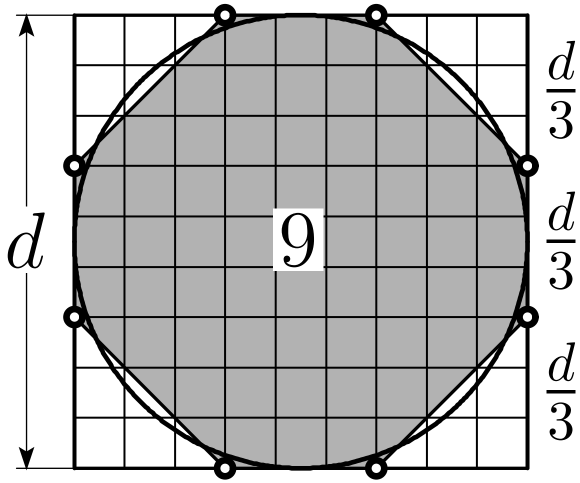




Area of the Circle.

Rhind Papyrus (Luxor, 19th century B.C.), Art. 48:





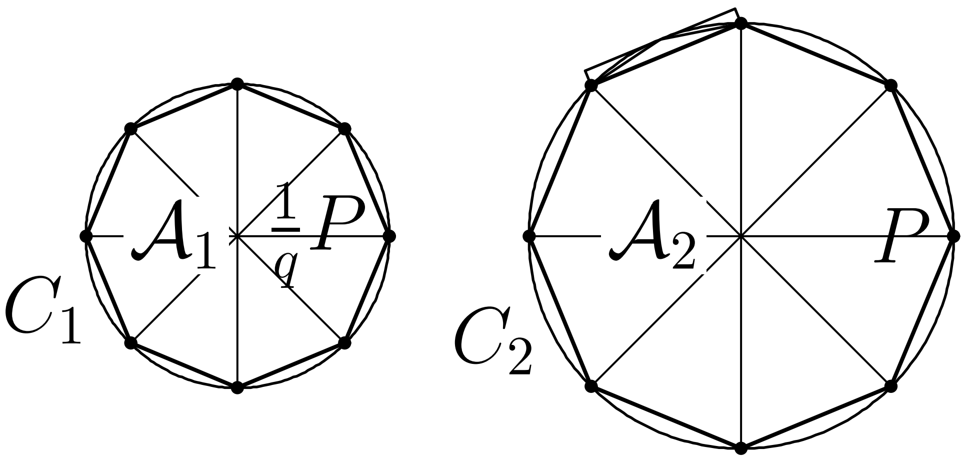
Square of $9 \times 9 = 81$ units;

"Circle" of $81 - 18 = 63 \approx 64$ uni

$64 = 8^2 = (9-1)^2$

"subtract $\frac{1}{9}$-th of diam. then square".

Euclid XII.2:



$$\frac{r_2}{r_1} = q \Rightarrow \frac{A_2}{A_1} = q^2.$$

hence $A = r^2\pi$

with unknown constant π.

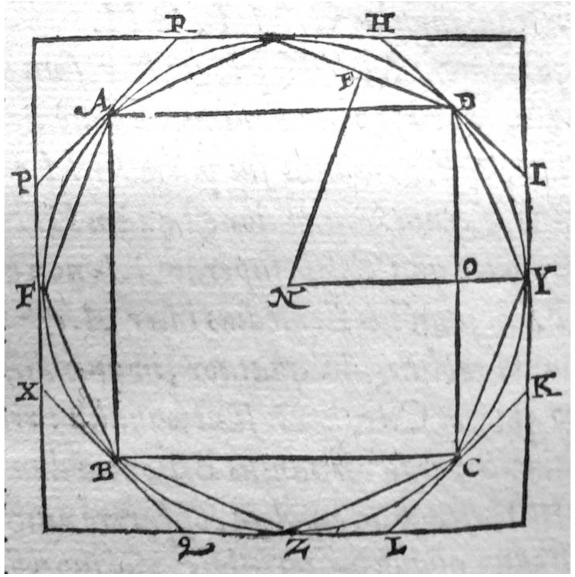
Egyptian: $\pi \approx \frac{256}{81} = 3.1605$

Archimedes’ Measuring of the Circle.

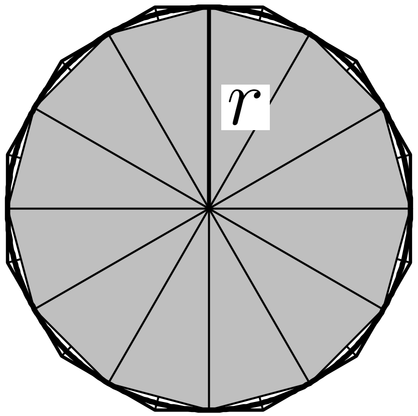
Proposition I:

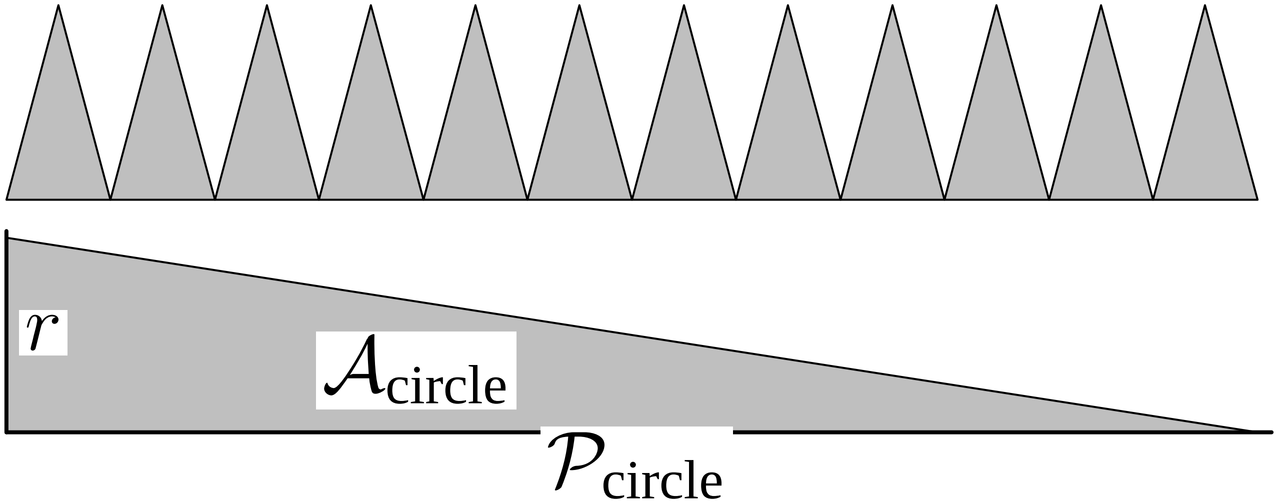
$$\mathcal{A}_{\text{circle}} = \mathcal{P}_{\text{circle}} \cdot \frac{r}{2}\,,$$

$$\Rightarrow\ \mathcal{P}_{\text{circle}} = 2r\pi\,.$$



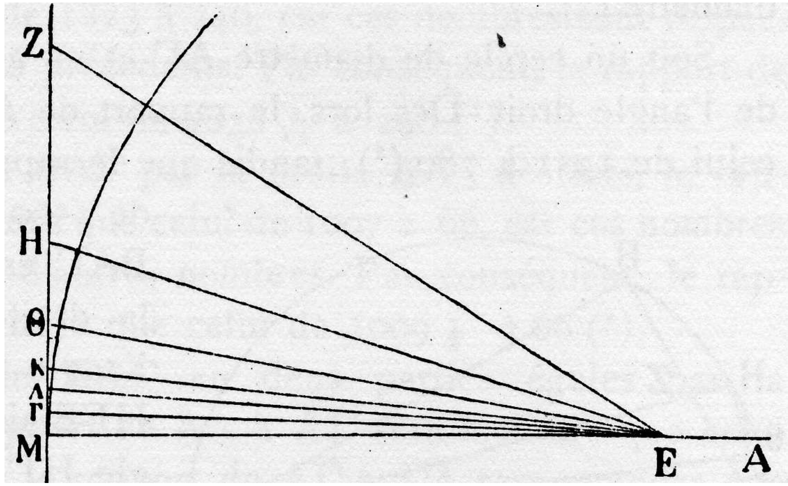
(Opera, 1615 (D. Rivault, BGE Ka459)

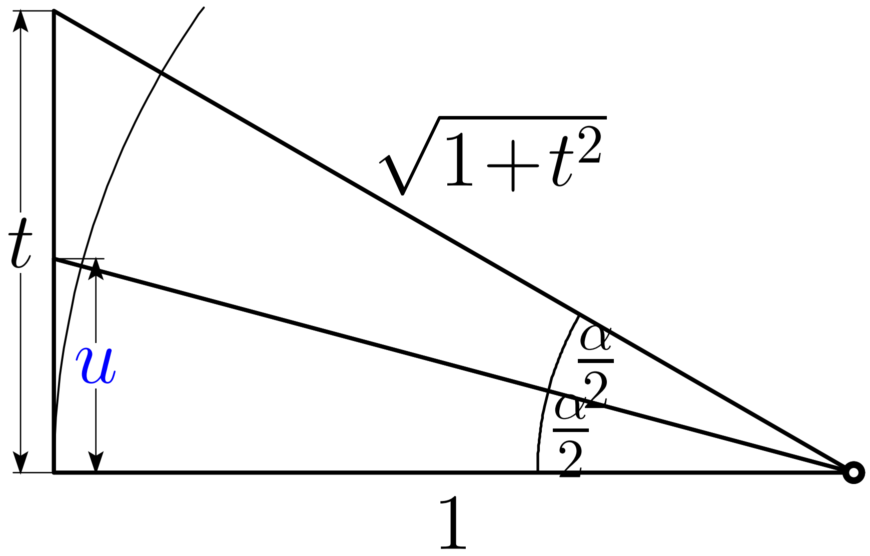




Upper bounds for π.

Eucl. VI.3: $$\frac{t-u}{\sqrt{1+t^2}} = \frac{u}{1} \quad \text{or} \quad u = \frac{t}{\sqrt{1+t^2}+1} \mapsto t\,.$$

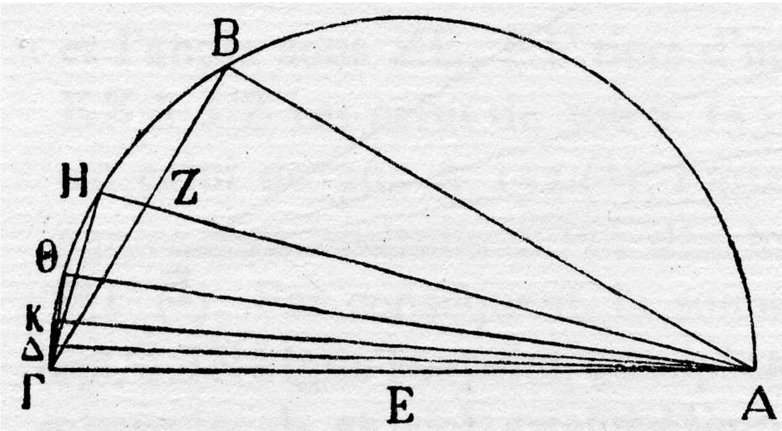


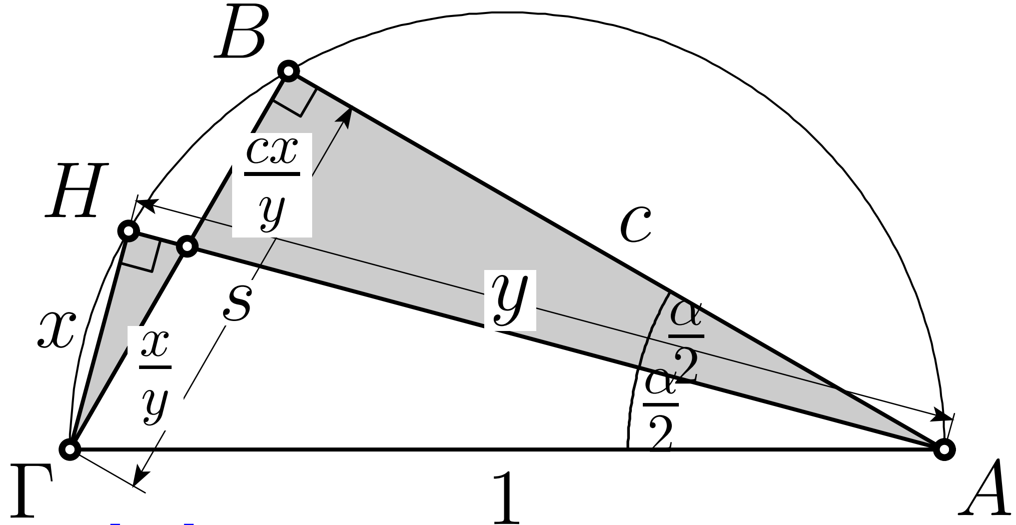


$$\Gamma Z < \frac{153}{265},\ \Gamma H < \frac{153}{571},\ \Gamma\Theta < \frac{153}{1162\frac{1}{8}},\ \Gamma K < \frac{153}{2334\frac{1}{4}},\ \Gamma\Lambda < \frac{153}{4673\frac{1}{2}}$$

Lower bounds for π.

Problem: Given s, c for α, find x, y for $\frac{\alpha}{2}$.





Must be clever to avoid two square roots to calculate:

$$s = \frac{x}{y} + \frac{cx}{y} \quad \Rightarrow \quad \frac{y^2 + x^2}{x^2} = \frac{1}{x^2} = \frac{c^2 + 2c + 1 + s^2}{s^2} = \frac{2c+2}{s^2}.$$

$$x = \sqrt{\frac{s^2}{2c+2}}, \qquad y = \frac{(1+c)x}{s}.$$

$$B\Gamma \geq \frac{1}{2},\ H\Gamma > \frac{780}{3013\frac{3}{4}},\ \Theta\Gamma > \frac{240}{1838\frac{9}{11}},\ K\Gamma > \frac{66}{1009\frac{1}{6}},\ \Lambda\Gamma > \frac{66}{2017\frac{1}{4}}.$$

PROPOSITION III:

$$3\,\frac{10}{71} < \pi < 3\,\frac{1}{7}$$

"Trust is good, control is better":

n	t_{corr}	t_{arch}	$\pi <$	s_{corr}	s_{arch}	$\pi >$
6	0.577350	0.577358	3.464102	0.500000	0.500000	3.000000
12	0.267949	0.267951	3.215390	0.258819	0.258814	3.105829
24	0.131652	0.131655	3.159660	0.130526	0.130519	3.132629
48	0.065543	0.065546	3.146086	0.065403	0.065400	3.139350
96	0.032737	0.032738	3.142715	0.032719	0.032718	3.141032

PROPOSITION III:

$$3\,\frac{10}{71} < \pi < 3\,\frac{1}{7}$$

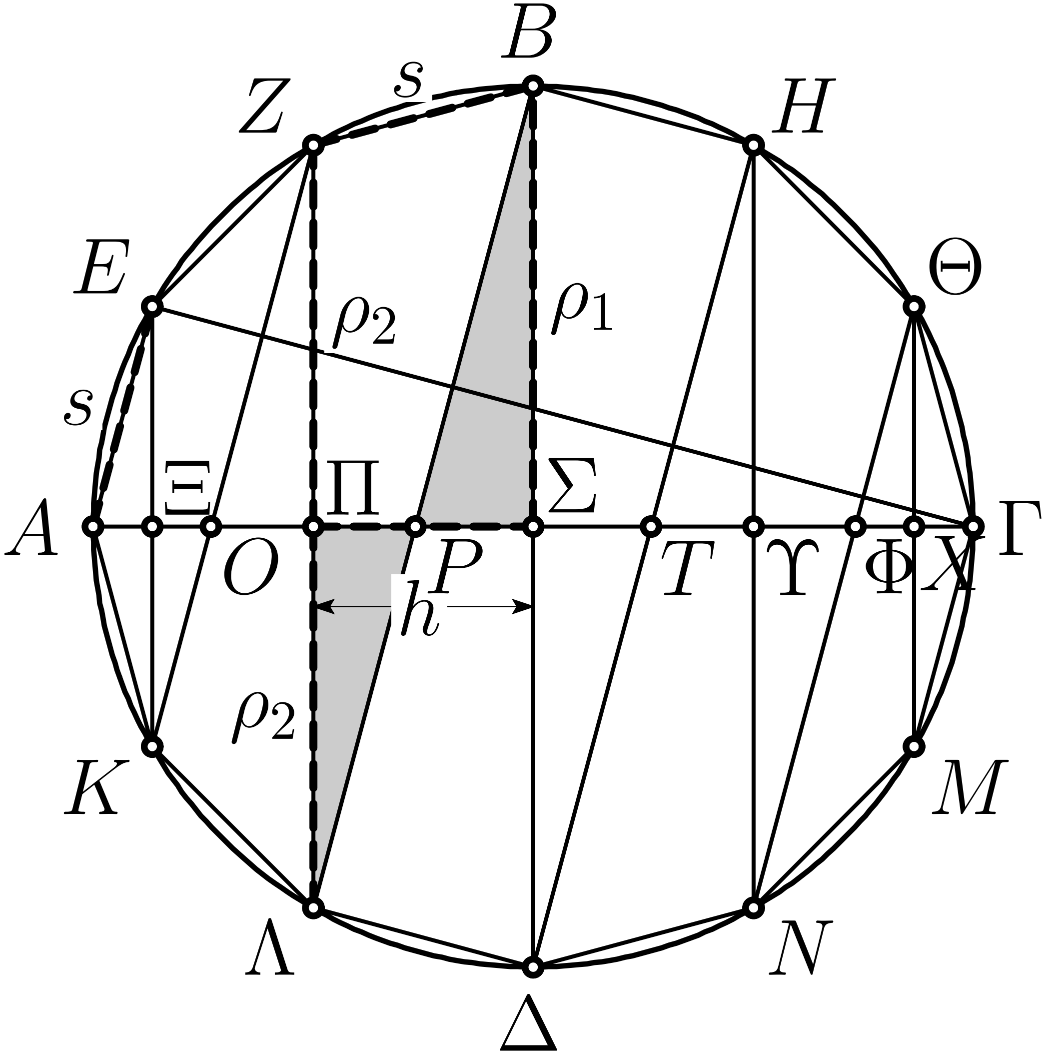
"Trust is good, control is better":

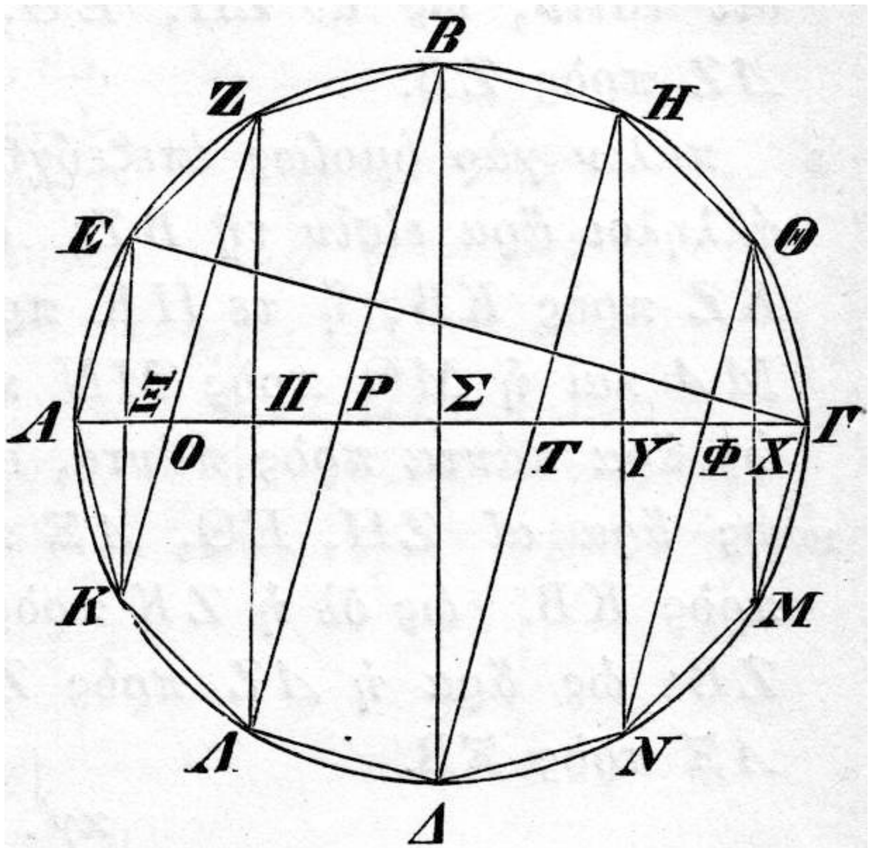
n	t_{corr}	t_{arch}	$\pi <$	s_{corr}	s_{arch}	$\pi >$
6	0.577350	0.577358	3.464102	0.500000	0.500000	3.000000
12	0.267949	0.267951	3.215390	0.258819	0.258814	3.105829
24	0.131652	0.131655	3.159660	0.130526	0.130519	3.132629
48	0.065543	0.065546	3.146086	0.065403	0.065400	3.139350
96	0.032737	0.032738	3.142715	0.032719	0.032718	3.141032

P.S. This control detected no error in Archimedes, but a printing error in the first edition of our book ...

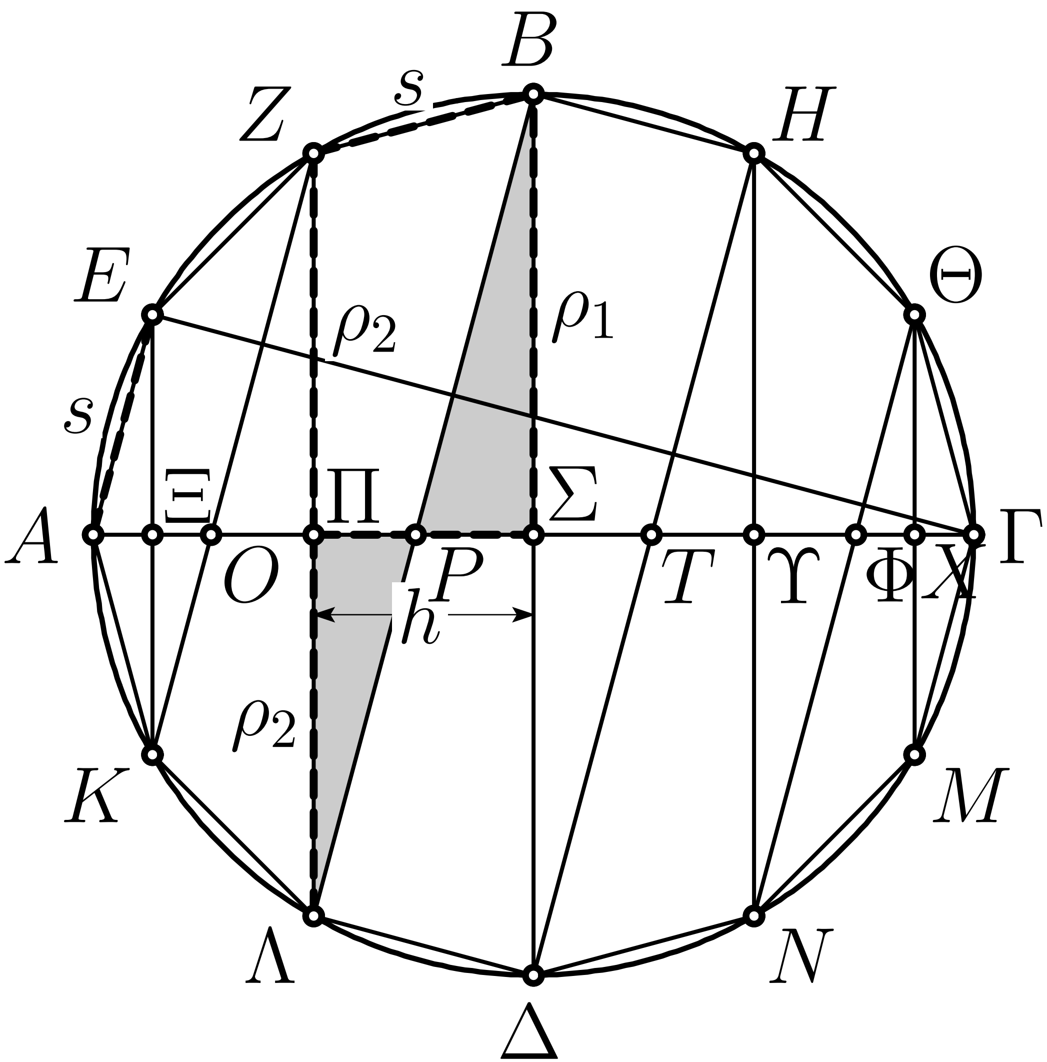
DE SPHAERA ET CYLINDRO (en total 229 pages).

"… since any circle equals a triangle whose basis is the perimeter and whose height is the radius, I had the intuition that any sphere equals a cone whose basis equals the **surface** of the sphere, and the height is the radius."
(Archimedes, in *The Method*, see Ver Eecke (1960), p. 488)





Opera ed. Heiberg 1910, p.89, BGE FL10626



Eucl. III.21: similar triangles

$$h = P\Sigma + \Pi P = \frac{(\rho_1 + \rho_2)s}{\Gamma E}\,.$$

Rotate figure around $A\Gamma$:

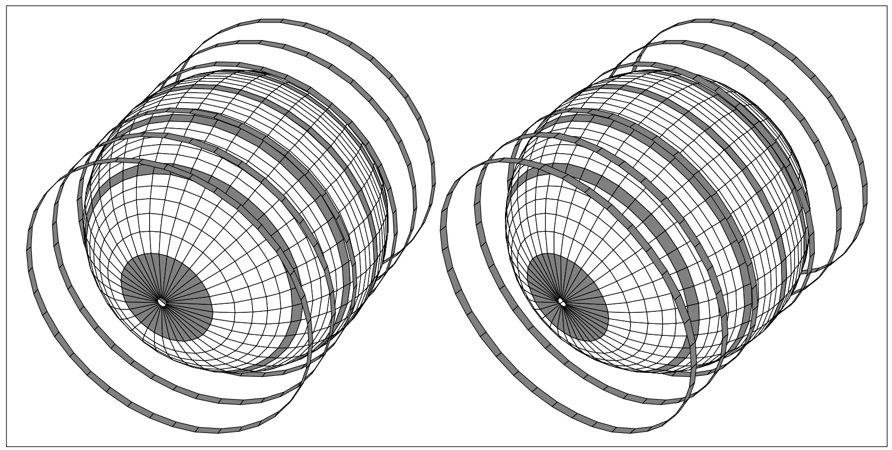
$$\mathcal{A}_{\text{ribbon}} = \frac{2\pi\rho_1 + 2\pi\rho_2}{2} \cdot s$$

$$= h \cdot \Gamma E \cdot \pi \to h \cdot 2r\pi\,, \quad \Rightarrow$$

cylinder projection preserves area

Prop. XXXIII: $\mathcal{A}_{\text{sphere}} = 4r^2\pi$

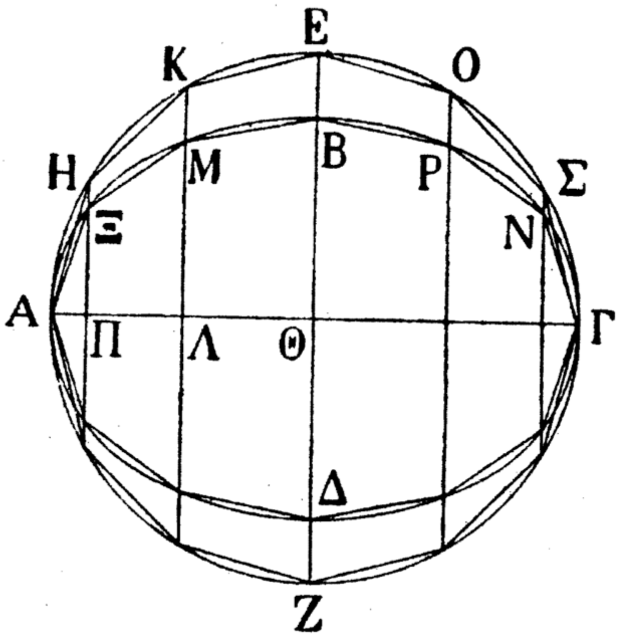
Prop. XXXIV: $\mathcal{V}_{\text{sphere}} = \frac{4r^3\pi}{3}\,.$



DE CONOIDIBUS ET SPHAEROIDIBUS. (3rd proof!)

"But afterwards, when I had studied them with greater care, I discovered what I had failed in before." (from the Introduction).

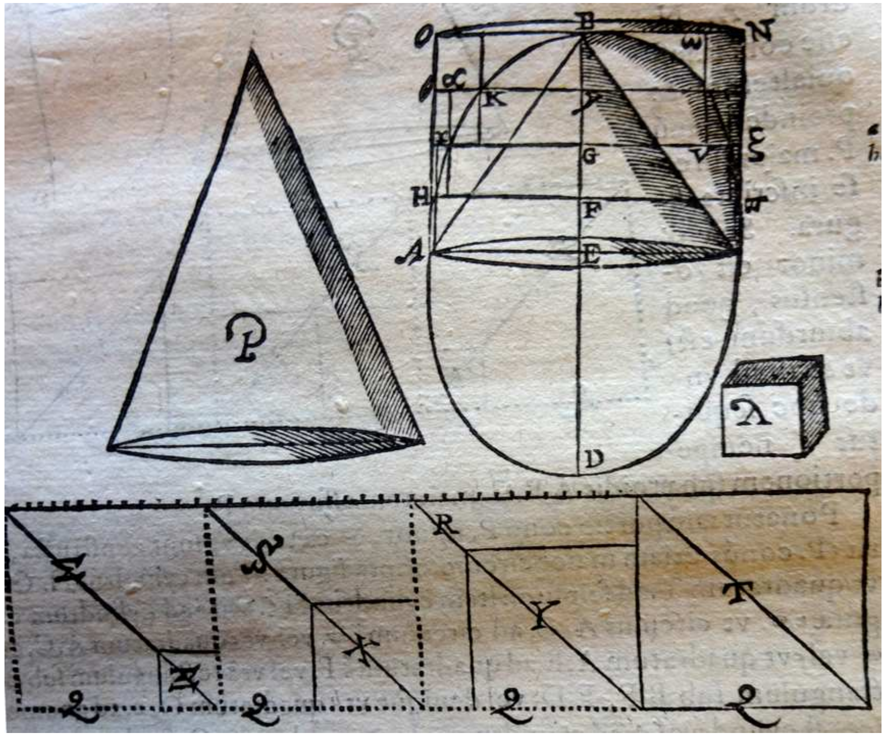
Proposition IV



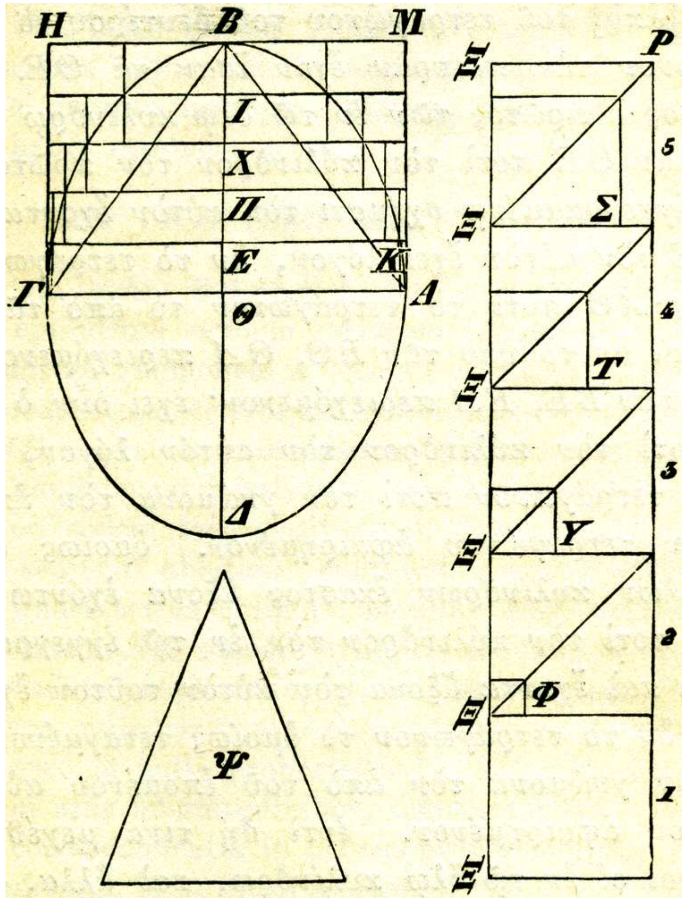
(Oeuvres, Ver Eecke 1960)

(Area of Ellipse by Trap. Rule)

Proposition XXVII



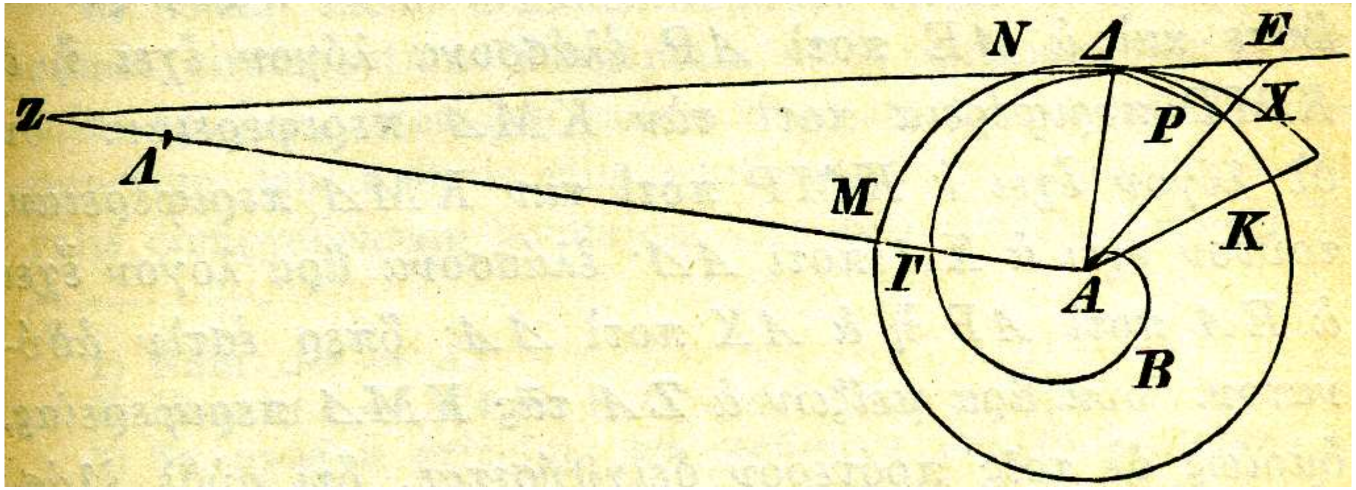
(Opera, Rivault 1615)

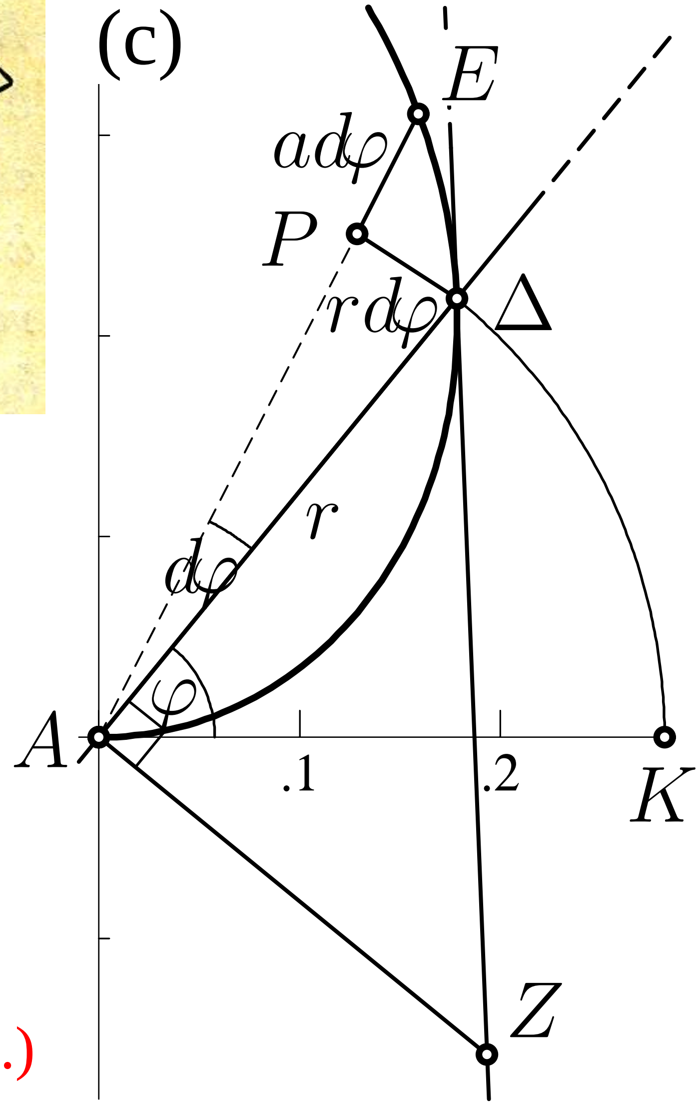


(Opera, Heiberg 1910)

(Vol. of Ellipsoid by upper and lower Riemann sums)

DE LINEIS SPIRALIBUS. $r = a\varphi$.

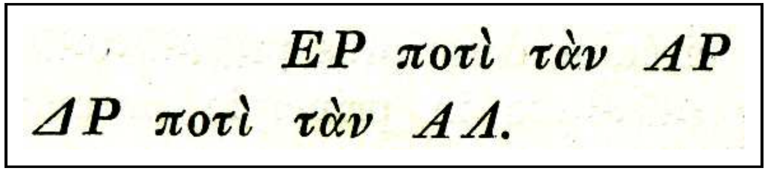




Proposition XX:

ΔZ = tangent in Δ if AZ = arc ΔK.

Proof. Archimedes: $\frac{EP}{AP} = \frac{\Delta P}{AZ}$;



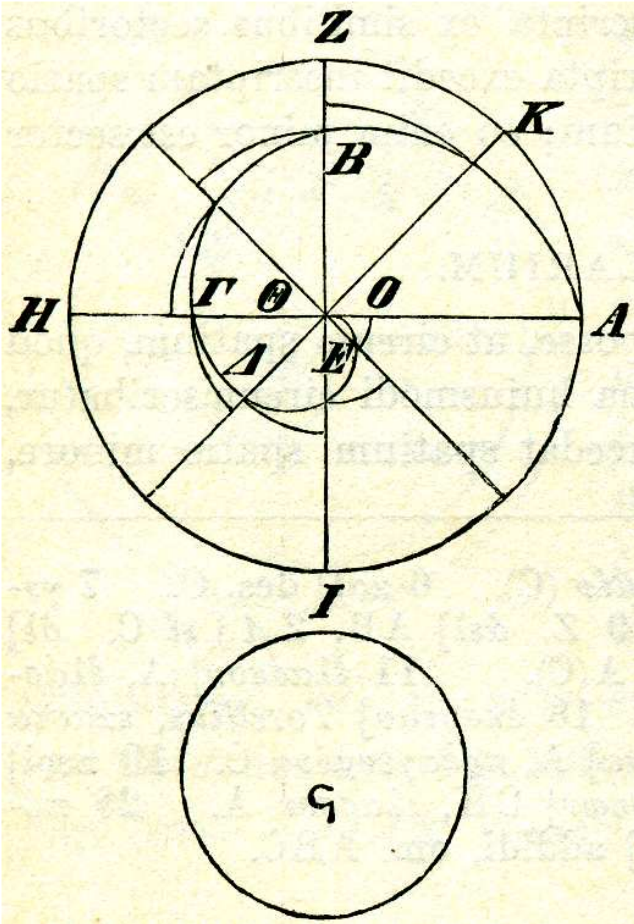
EP ποτὶ τὰν AP ΔP ποτὶ τὰν AΛ.

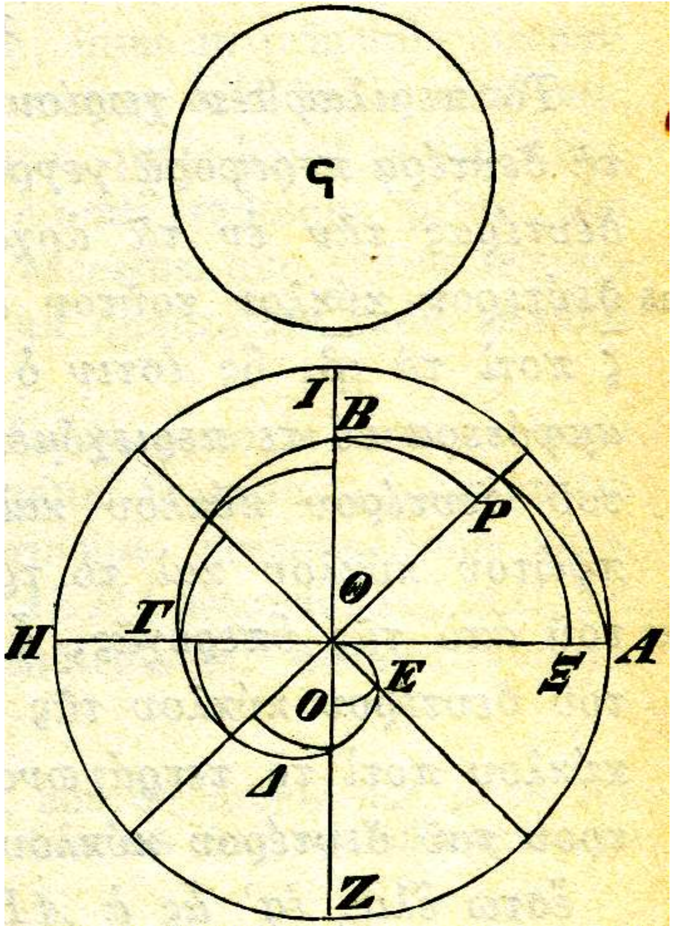
(in fact more carefully: choose a $\Lambda \neq Z \Rightarrow$ contradict.)

Leibniz: $\frac{ad\varphi}{r} = \frac{rd\varphi}{AZ}$ or $AZ = \frac{r^2}{a} = r\varphi$. $\square$

Proposition XXIV. The area of the spiral (spatium comprehensum spirali ...) after one revolution (prima circumactione ...) is $\frac{1}{3}$-rd (tertia pars ...) of the area of the circle.

Proof. Careful estimations ...





by upper ... ... and lower Riemann sums.

(P.S. He quotes this result in his *sphaeroidibus*.)

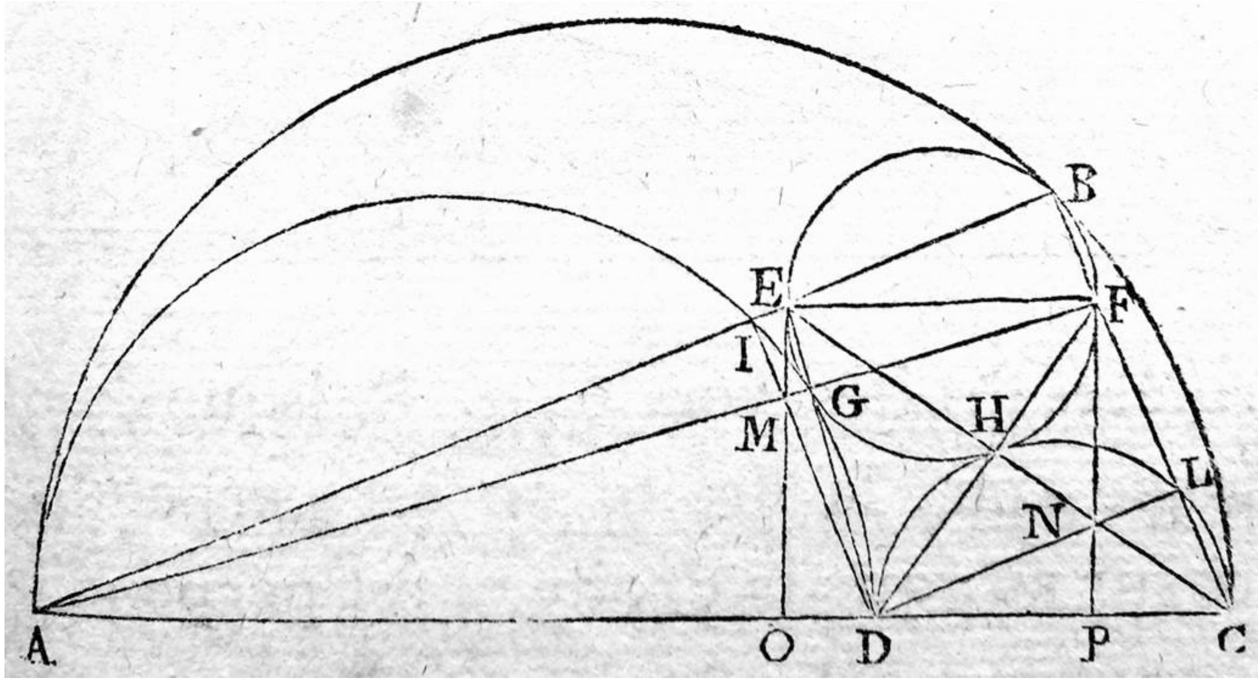
Book of Lemmas.

(Greek text is lost; survived through Arabic translation by Thābit ibn Qurra)

Prop. VI.

A, O, P, C in continued proportion

or $AO : OP = OP : PC$.



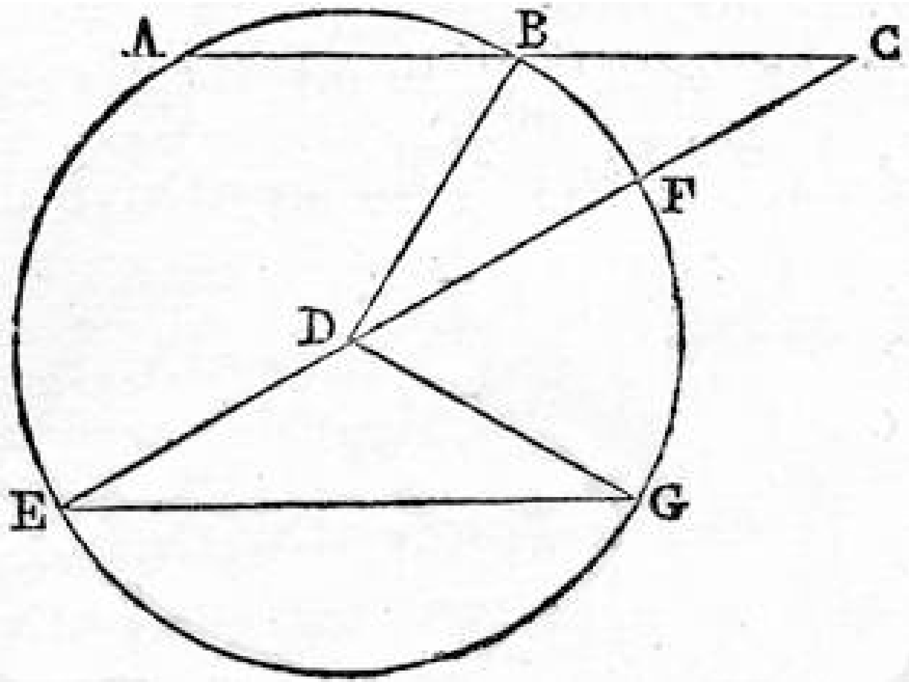
(copied from Peyrard's edition of Archimedes' *Opera*, vol. 2, Paris 1808)

(come back to this in talk on Steiner.)

Book of Lemmas contd.

Prop. VIII. If $BC = BD$,

arc $AE = 3$ arc BF.

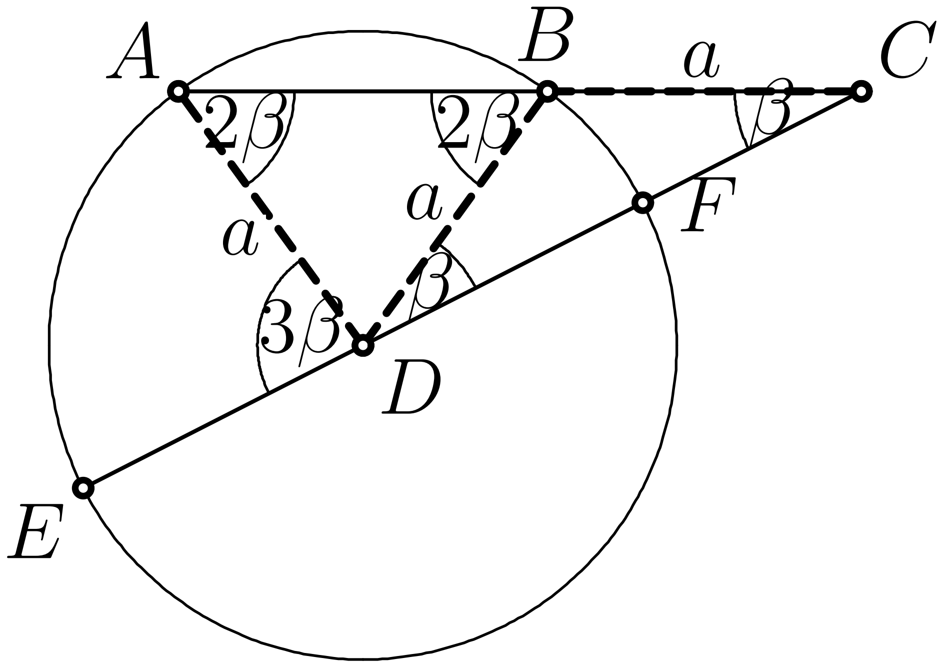


Proof. Draw parall. EG,

Eucl. III.20:

arc $FG = 2$ arc BF.

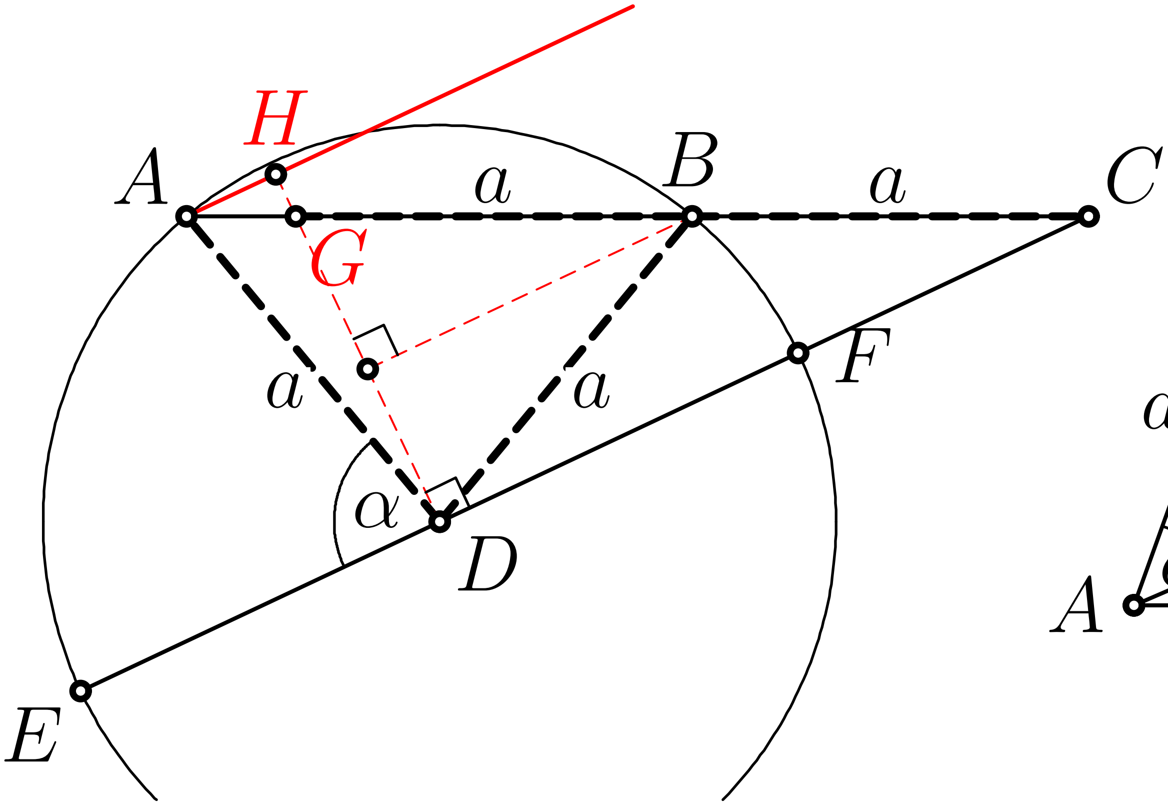
Simplified proof
by Viète's "ladder" (1593):

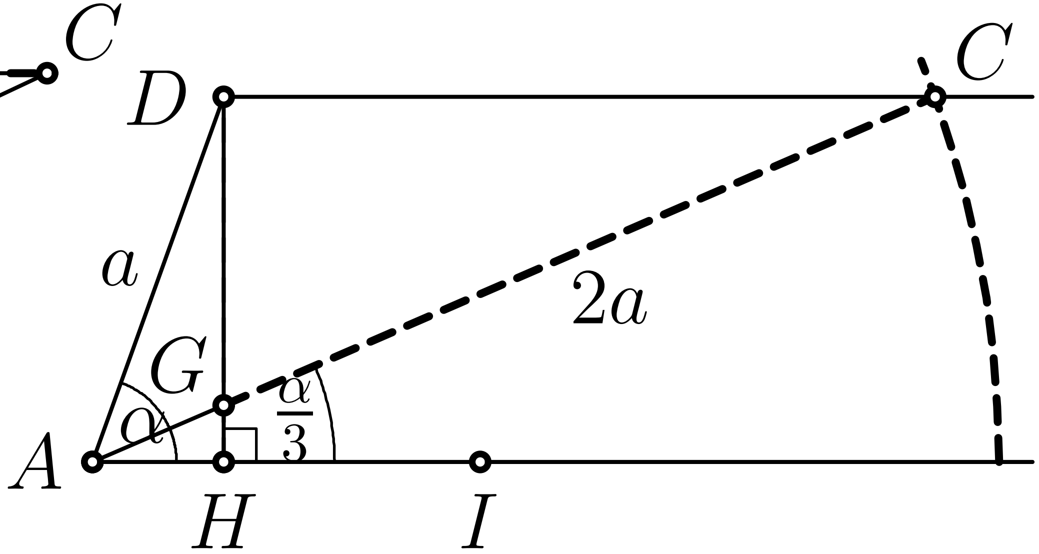


use alternatively
Eucl. I.5 and Eucl. I.32.

Pappus Book IV. Prop. 32:

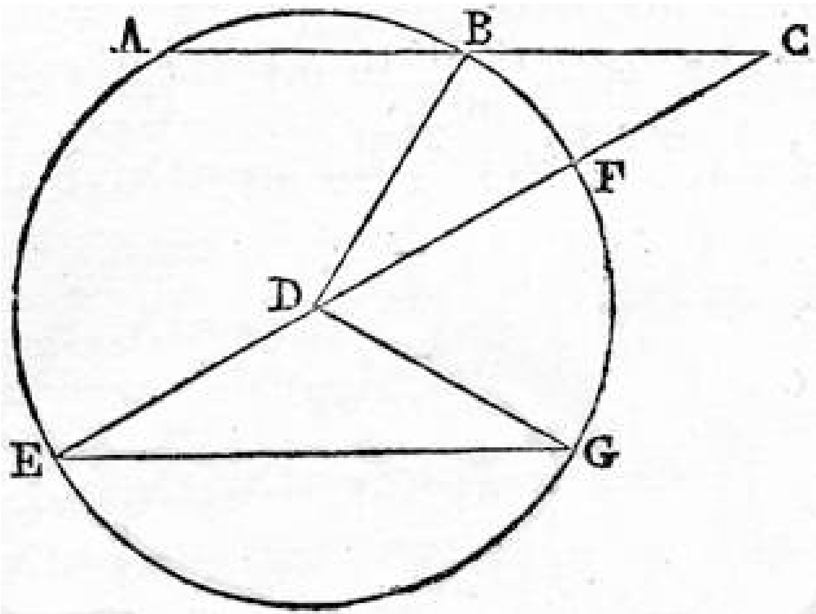
Draw parallel AH and perpendicular DH $\Rightarrow$





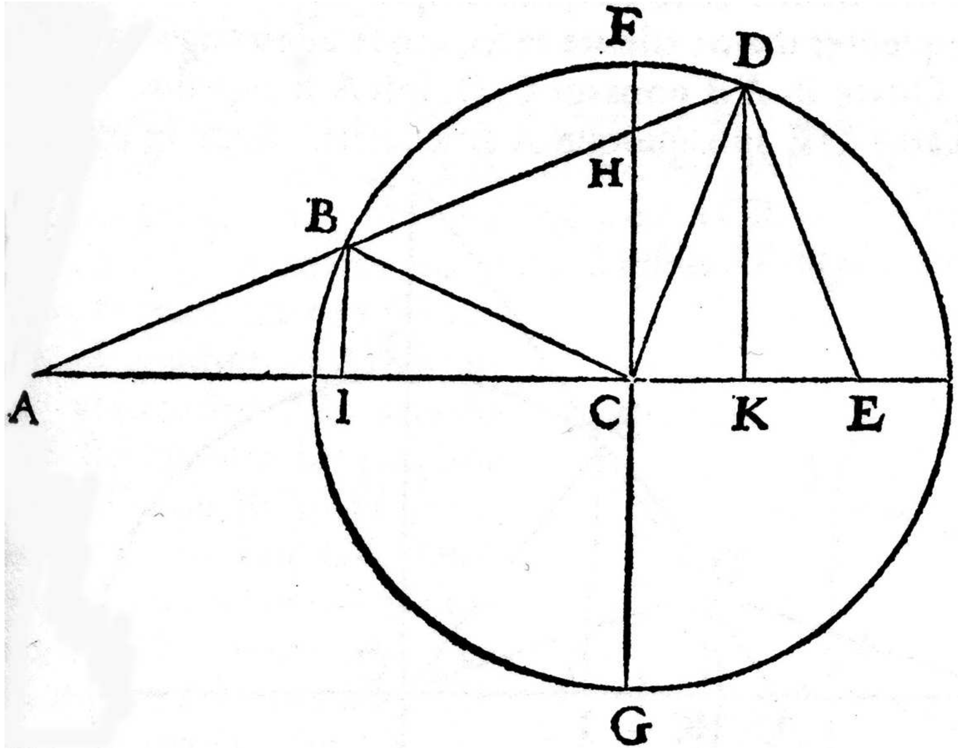
leads to angle-trisection algorithm using the **conchiod** ↑.

Viète, *Supplementum Geometriæ*, 1593:



Archimedes, Lemma 8

$\Rightarrow$



Viète 1593

known $CE = d = 2\cos 3\alpha$, have to find $AC = x = 2\cos\alpha$.

$CH^2 = 4 - x^2\,,$ (Pythagoras) $HD \cdot 1 = 1 - CH^2\,,$ (Eucl. III.35 for H) $\frac{d}{x} = \frac{HD}{1}\,.$ (Thales)

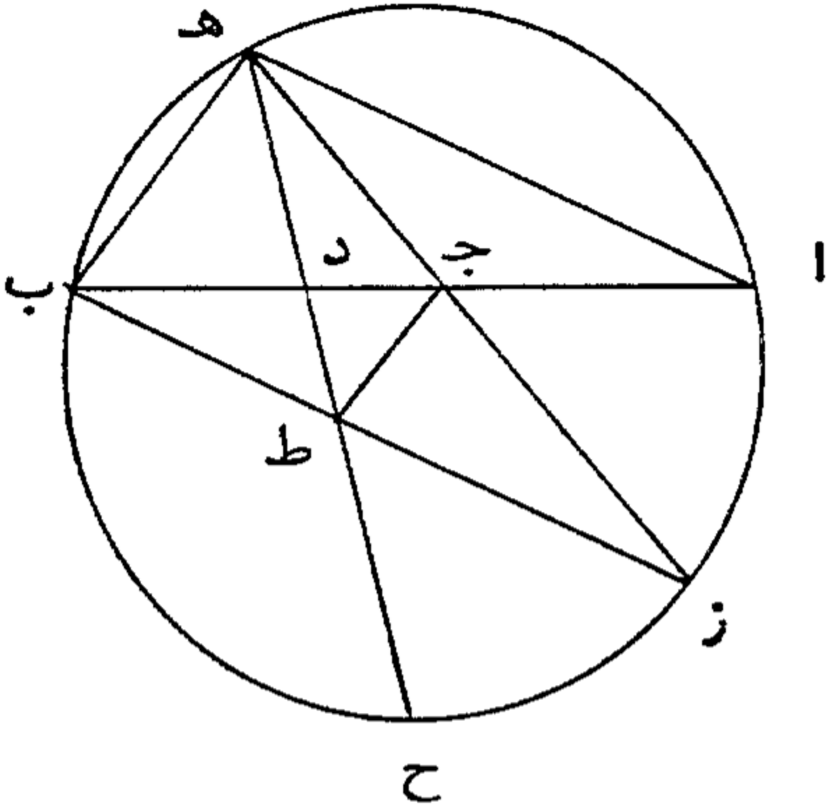
1 C — 3 N, æquatur 1.

$$\boxed{x^3 - 3x = d}$$

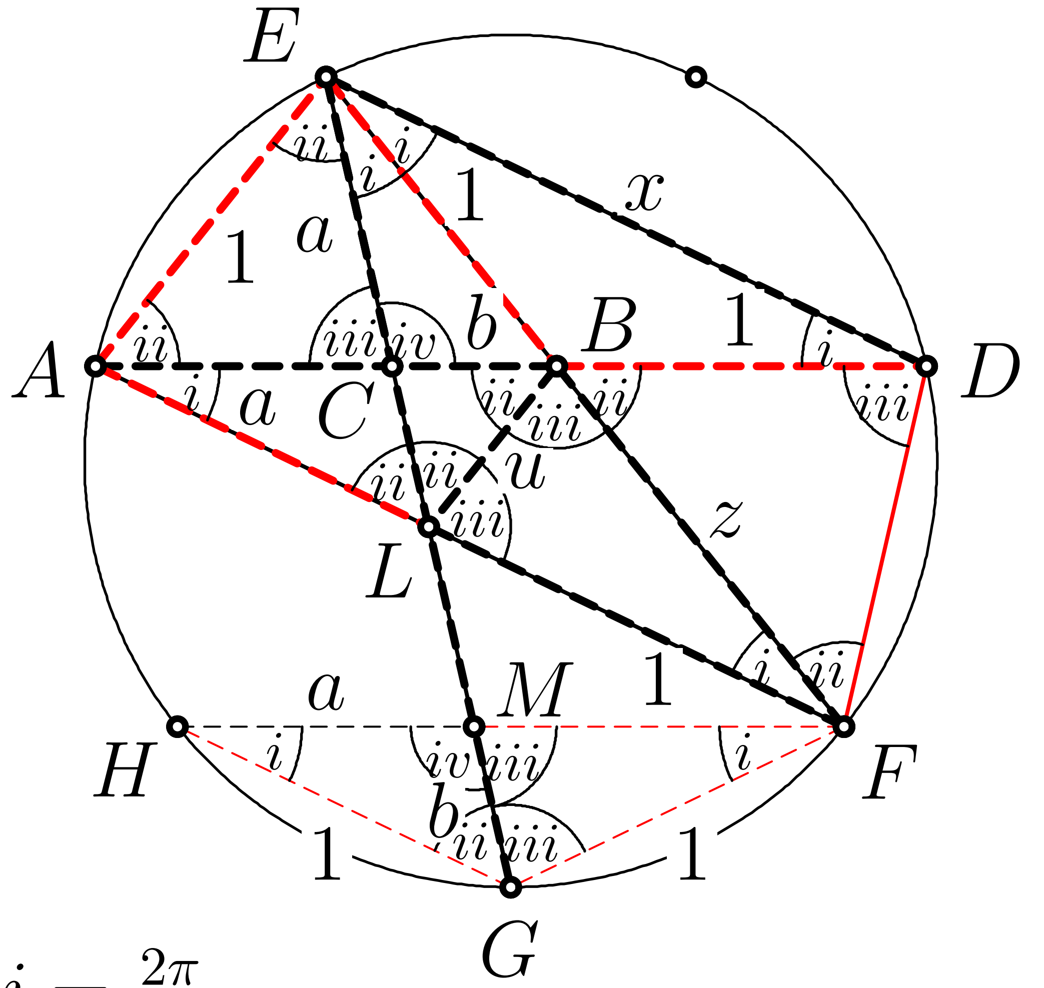
$\Rightarrow$ cubic equation for trisection

The Heptagon.

(Greek text lost; Arabic translation by Thābit ibn Qurra, engl. transl. and comments Hogendijk 1984, see also Paunić 2006)



Archimedes-Thābit-Hogendijk



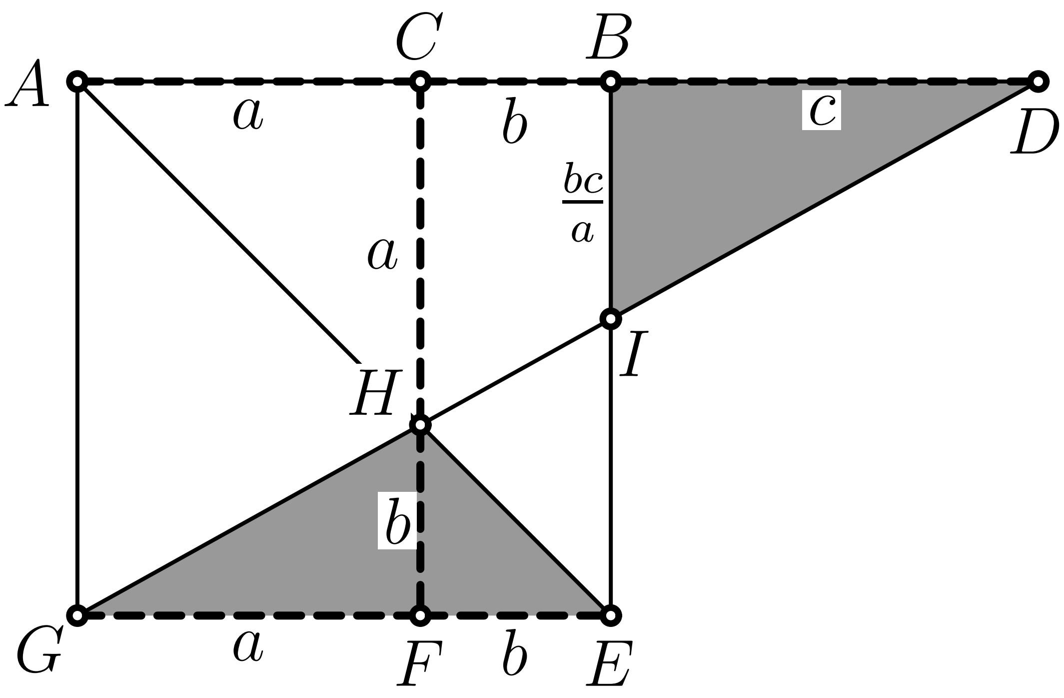
Eucl. III.21: all angles $i = \frac{\pi}{7}, ii = \frac{2\pi}{7}$...

$$\frac{EA}{AC} = \frac{AB}{EA} \quad \text{or} \quad BD^2 = AC \cdot AB \quad \text{or} \quad 1 = a(a+b)\,, \qquad (\text{Arch}_1)$$

$$\frac{EC}{CB} = \frac{CD}{EC} \quad \text{or} \quad AC^2 = CB \cdot CD \quad \text{or} \quad a^2 = b(b+1)\,. \qquad (\text{Arch}_2)$$

How did Archimedes solve this "oldest nonlinear system of algebraic equations of humanity" by νεύσεις (verging) ?

$$c^2 = a(a+b)\,, \qquad (\text{Arch}_1)$$
$$a^2 = b(b+c)\,. \qquad (\text{Arch}_2)$$



draw square $ABEG$ with diag. AE;

"verge" line $GHID$ such that grey triangles have same area;

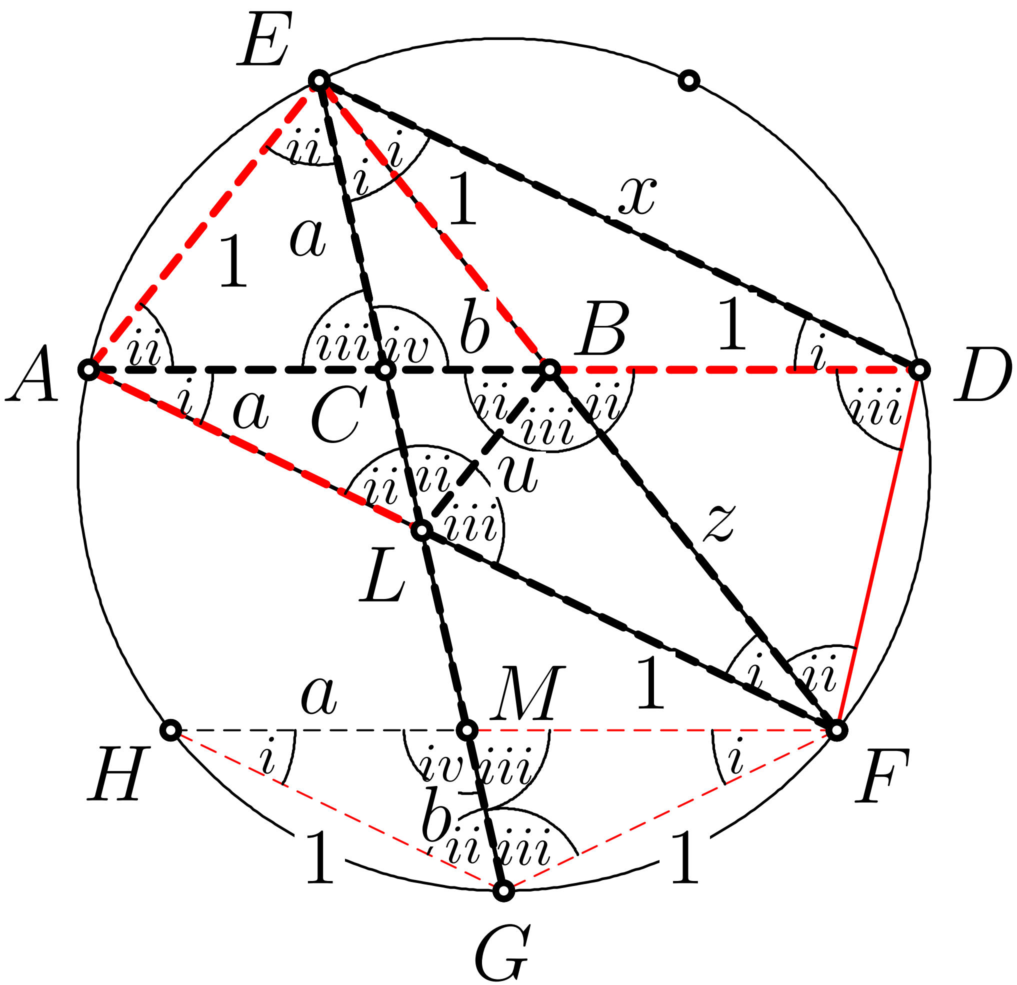
Thales $IBD \sim HFG \Rightarrow BI = \frac{bc}{a}$;

Thales $IBD \sim HCD \Rightarrow (\text{Arch}_2)$,

areas $IBD = CHE \Rightarrow (\text{Arch}_1)$.

More algebra allows to exploit Archimedes' picture further:

$$y = z+1\,,\; z = \frac{y}{x}\,,\; a+1 = x\,,\; b+1 = \frac{x}{z}\,,\; a = \frac{1}{z}\,,\; u = \frac{1}{x}\,,\; b = \frac{1}{y}\,.$$



$$y^3 - 2y^2 - y + 1 = 0$$
$$x^3 - x^2 - 2x + 1 = 0$$
$$a^3 + 2a^2 - a - 1 = 0$$
$$b^3 - b^2 - 2b + 1 = 0$$
$$z^3 + z^2 - 2z - 1 = 0$$
$$u^3 - 2u^2 - u + 1 = 0$$

1 C + 1 Q − 2 N, æquatur 1.

1 Q − 1 C, æquatur 1 + 2 N.

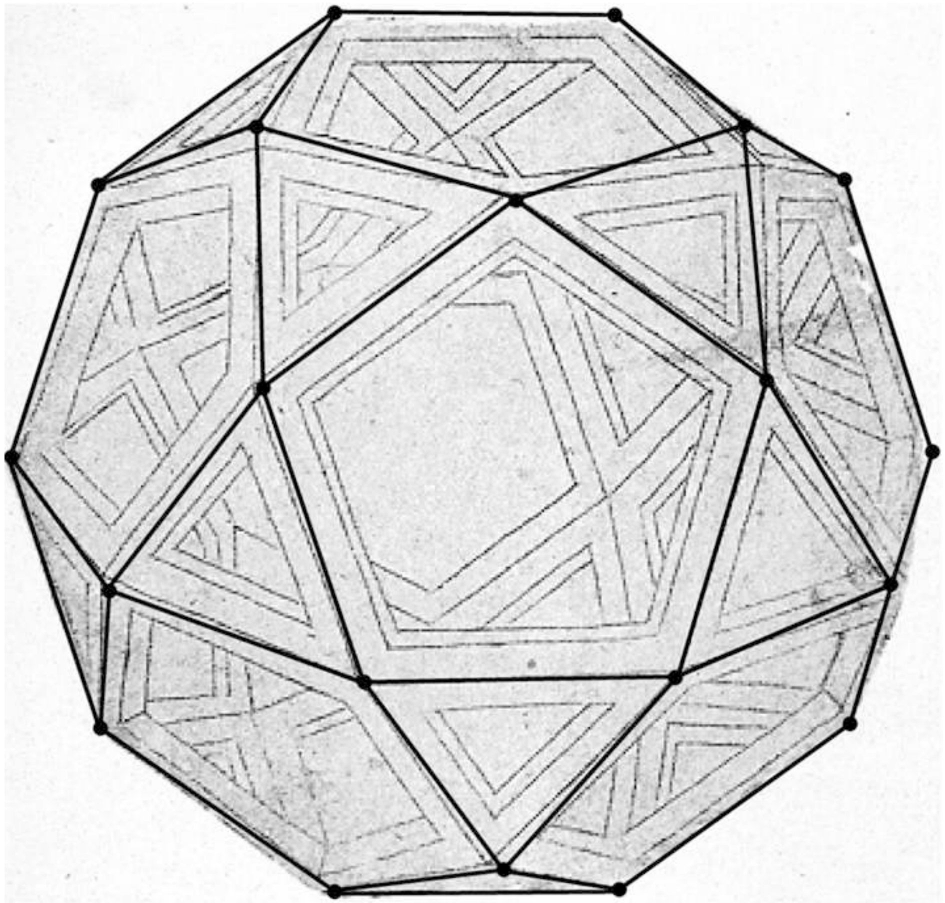
1 C − 1 Q, æquabitur 2 N − 1.

(the last three are the equations for z, b, x in Viète 1593)

In Archimedis solidorum doctrinam.

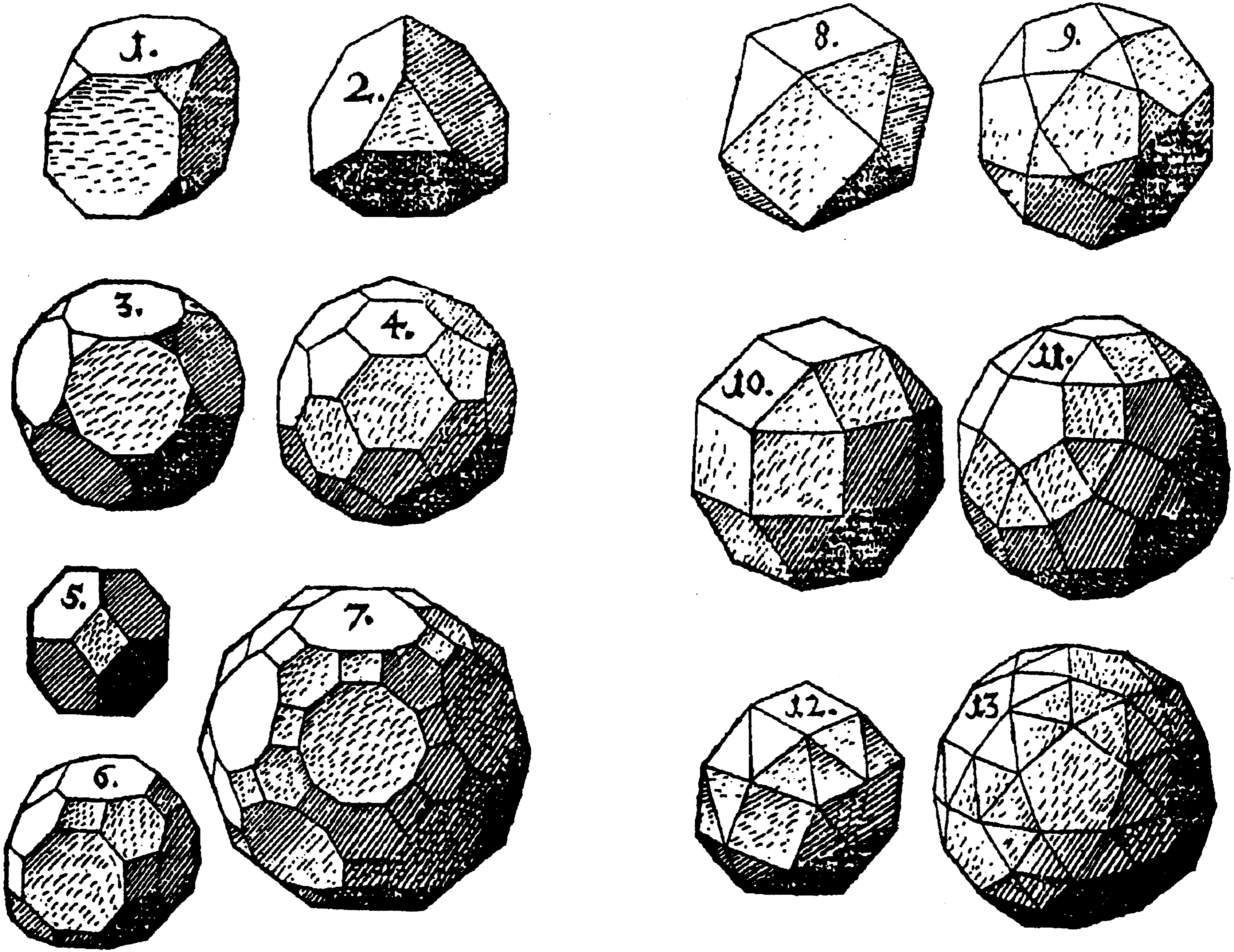
Pappus, in his *Collection*, Book V, gives a list of

> ſed etiam quæ ab 'Archimede inuenta ſunt, numero tredecim;



Leonardo da Vinci, corr. by Bernard Gisin.

Archimedes' text is lost; slow re-discovery over centuries (see picture) until Kepler 1619 found them all ...

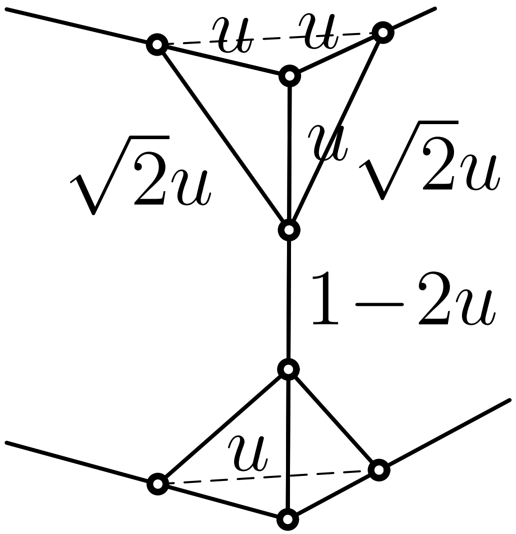


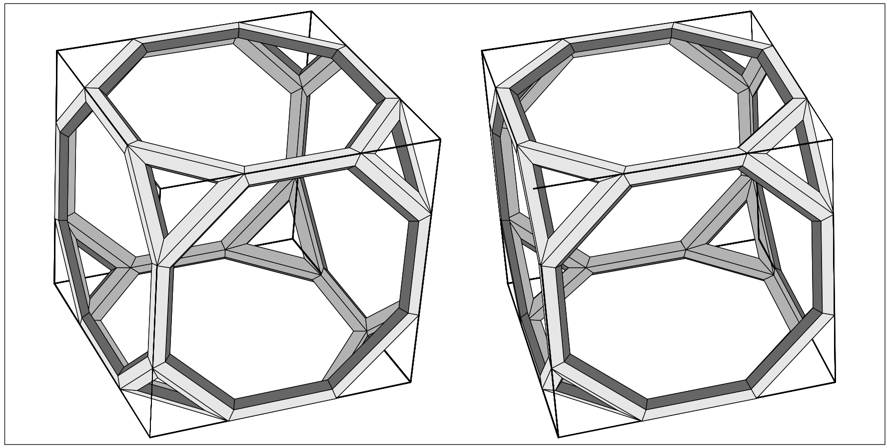
Kepler's "Archimedêa Corpora"

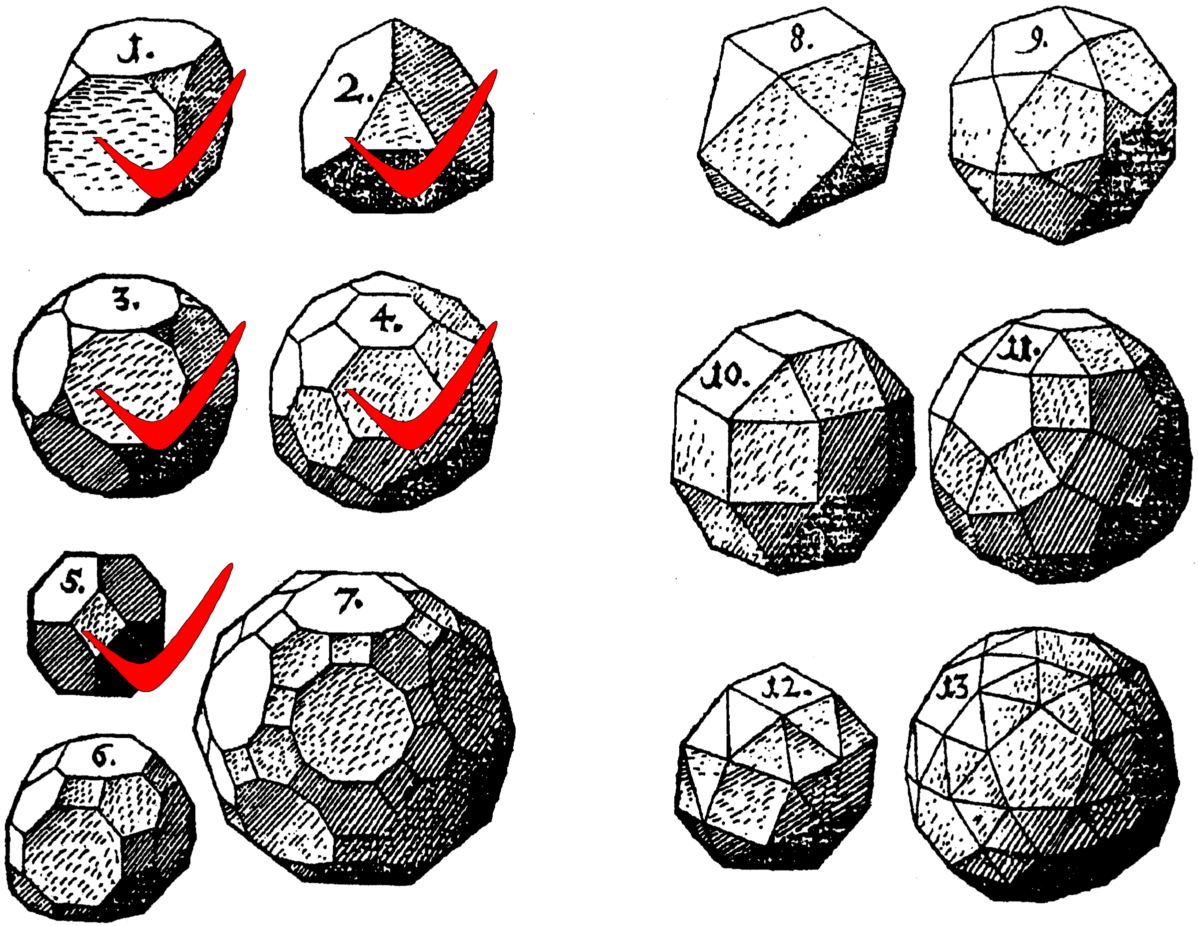
(*Harmonices mundi* 1619, Liber II, Propositio XXVIII).

... and gave these names:

1	trunc. cube	8 triang., 6 octog.	14	36	24
2	trunc. tetrahedron	4 triang., 4 hexag.	8	18	12
3	trunc. dodecahedron	20 triang., 12 decag.	32	90	60
4	trunc. icosahedron	12 pent., 20 hexag.	32	90	60
5	trunc. octahedron	6 squares, 8 hexag.	14	36	24
6	trunc. cuboctahedron	12 squares, 8 hexag., 6 octog.	26	72	48
7	trunc. icosidodecahedron	30 squares, 20 hexag., 12 decag.	62	180	120
8	cuboctahedron	8 triang., 6 squares	14	24	12
9	icosidodecahedron	20 triang., 12 pent.	32	60	30
10	rhombicuboctahedron	8 triang., 18 squares	26	48	24
11	rhombicosidodecahedron	20 triang., 30 squares, 12 pent.	62	120	60
12	snub cube	32 triang., 6 squares	38	60	24
13	snub dodecahedron	80 triang., 12 pent.	92	150	60

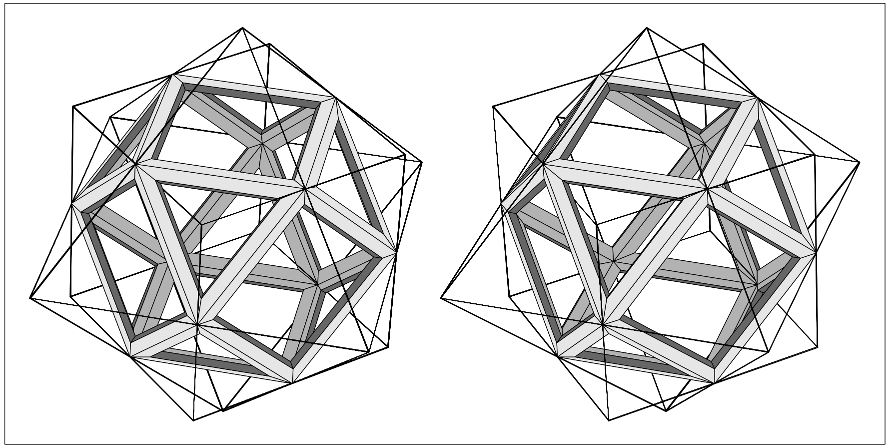


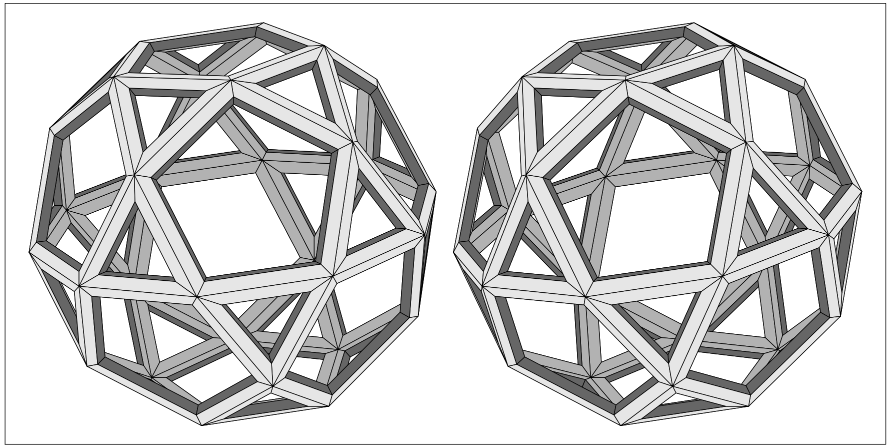




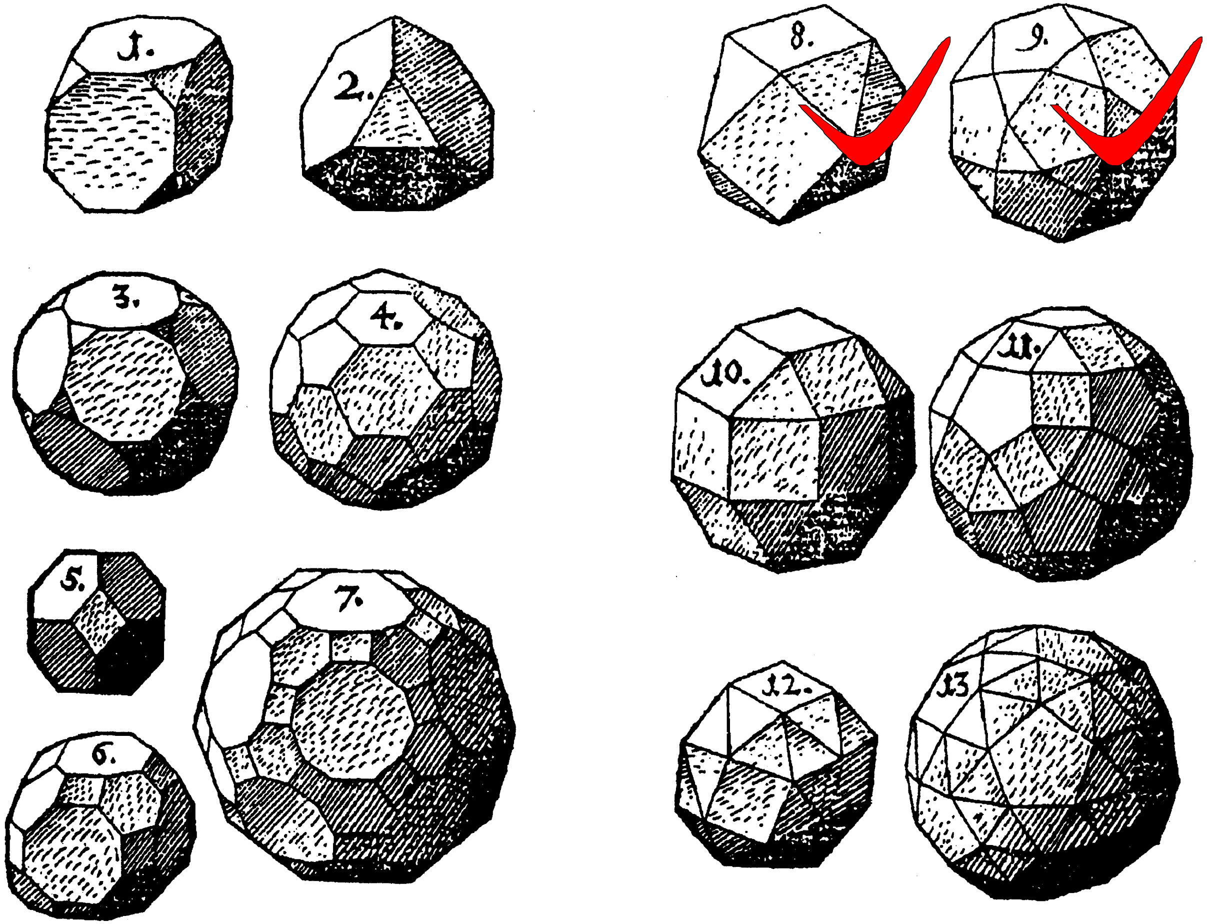
Construction of the Corpora from Plato's:

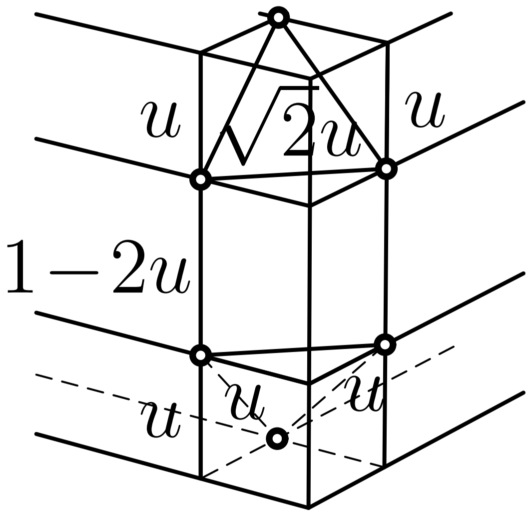
1. Truncating the vertices.

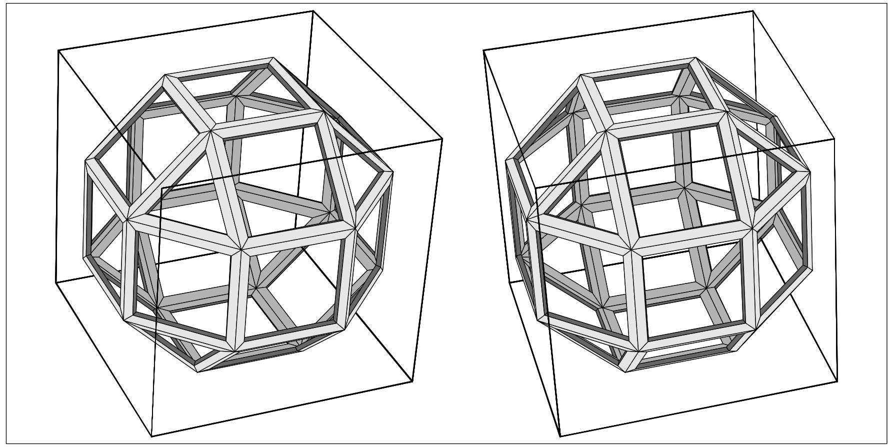




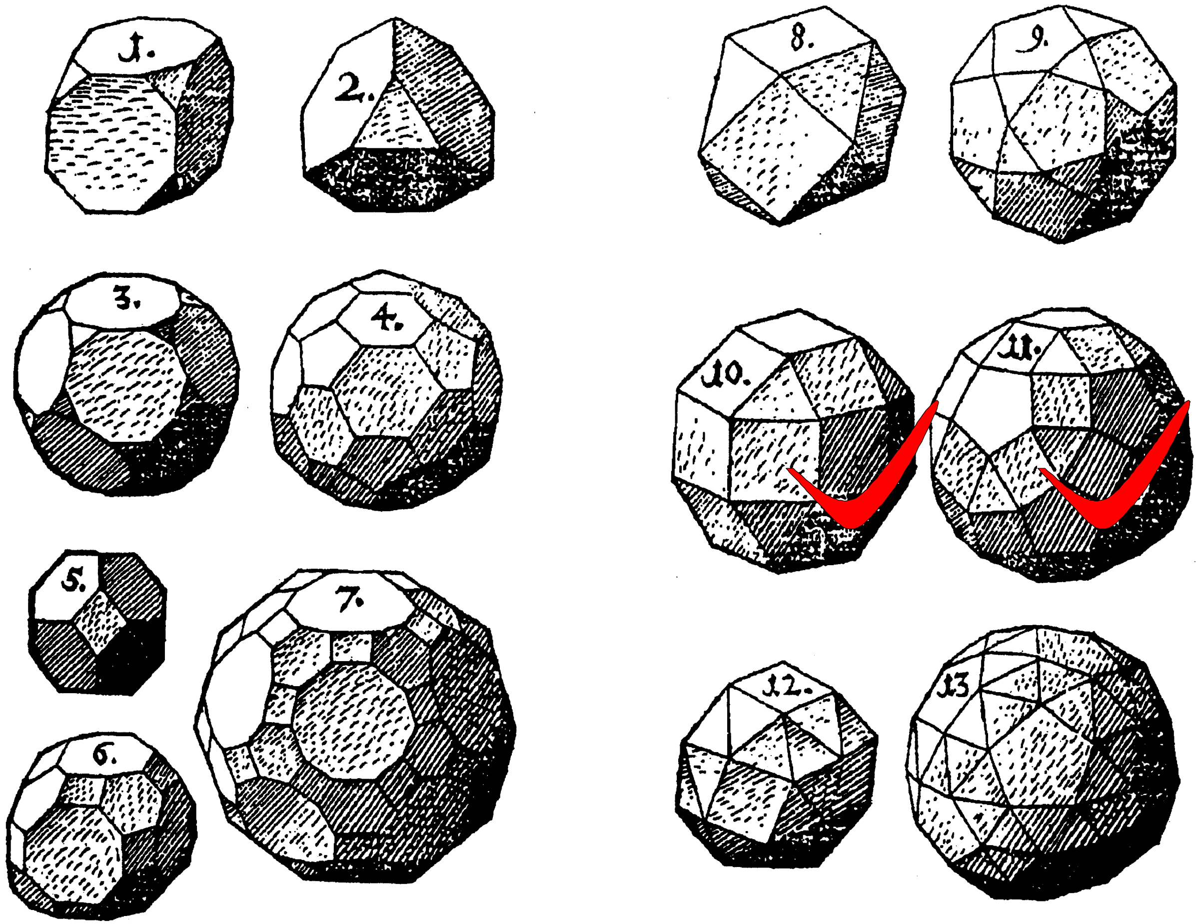
2. Truncating to the midpoint.

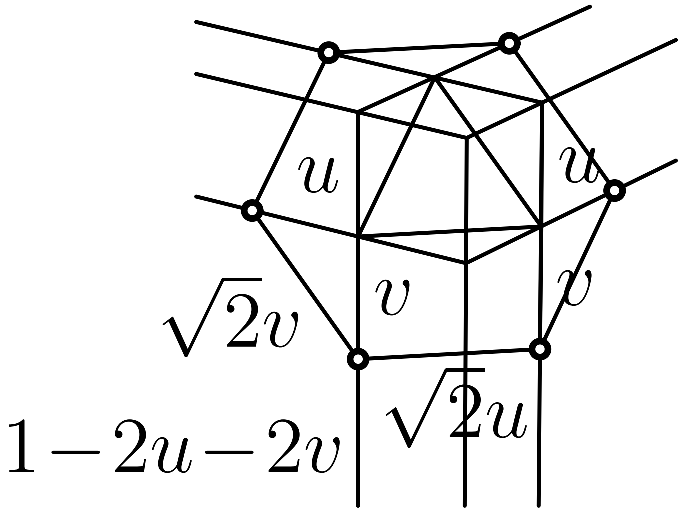


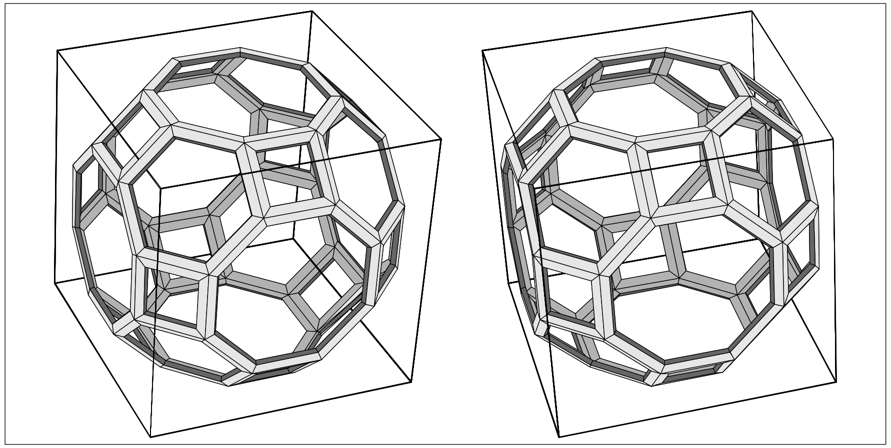


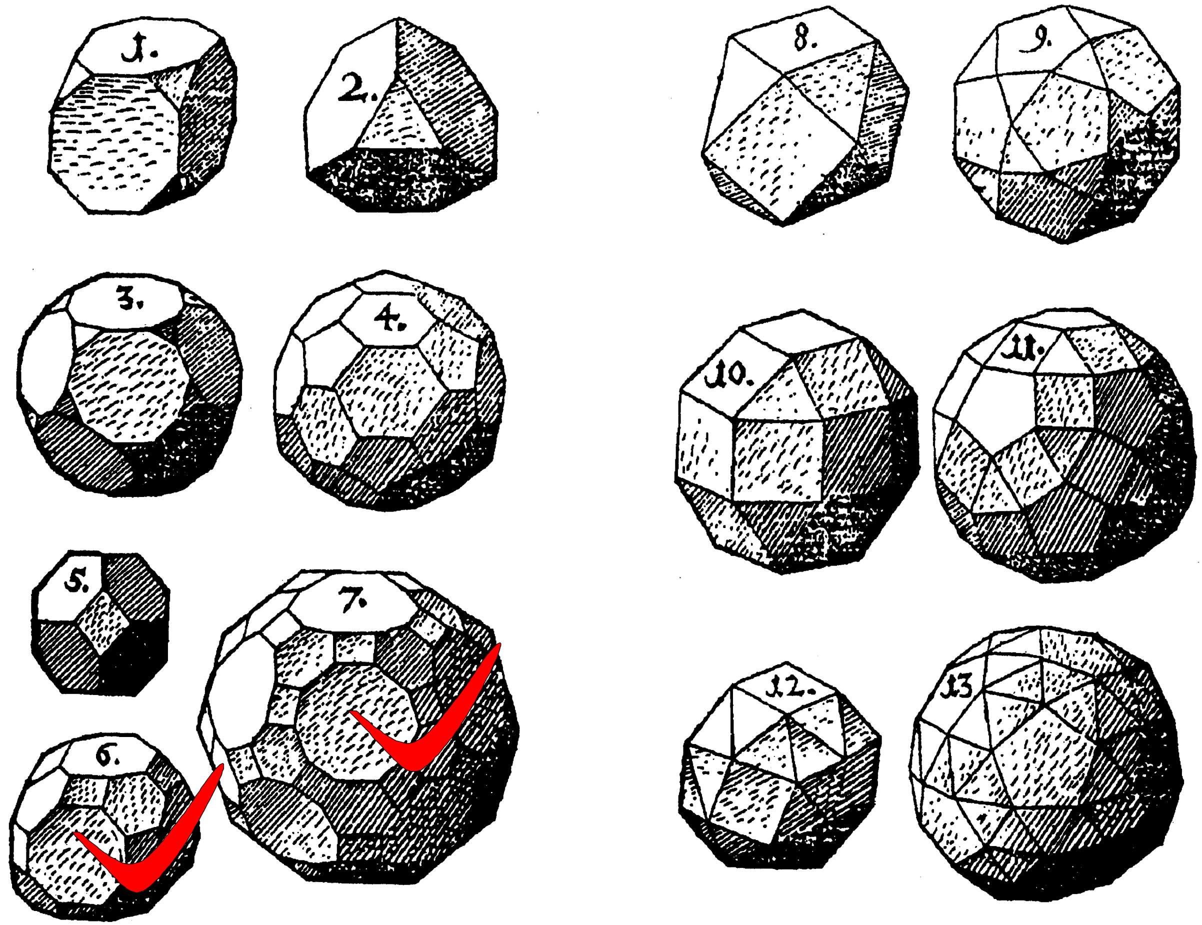


3. Truncating the edges.

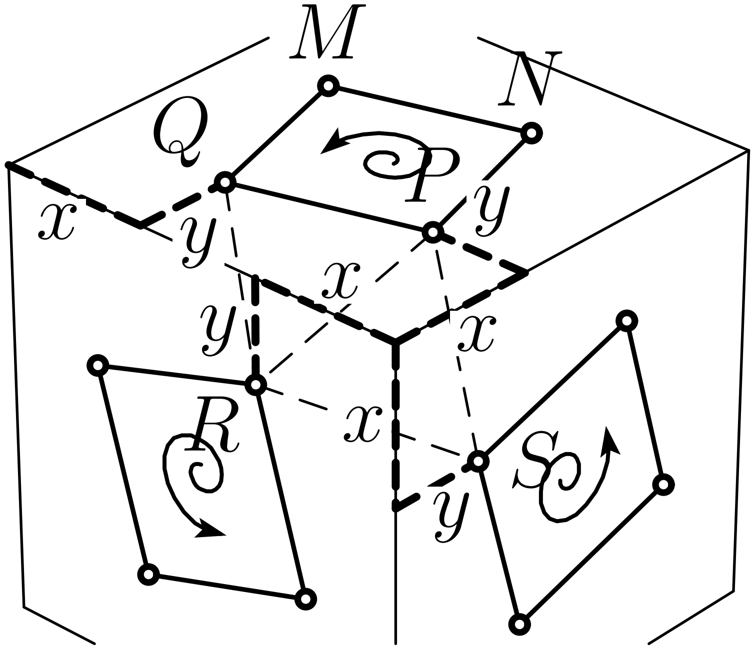


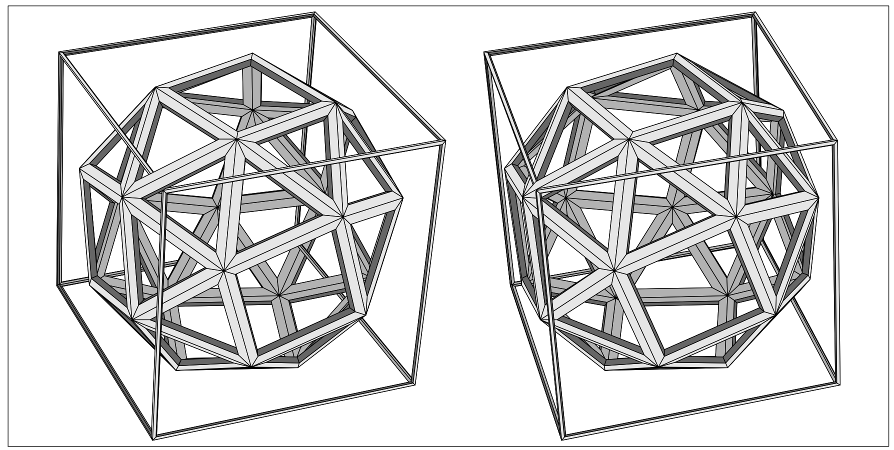




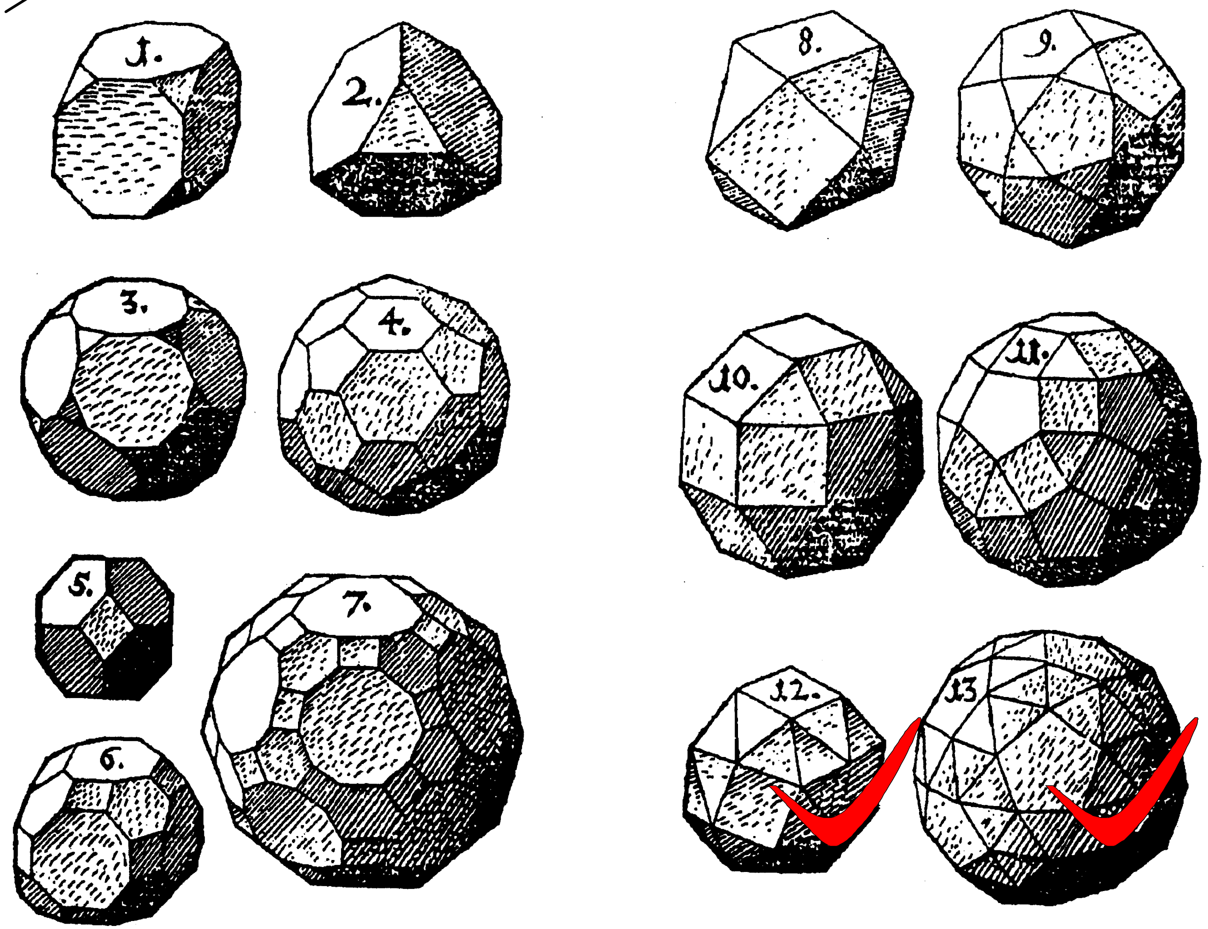


4. Truncating edges and vertices



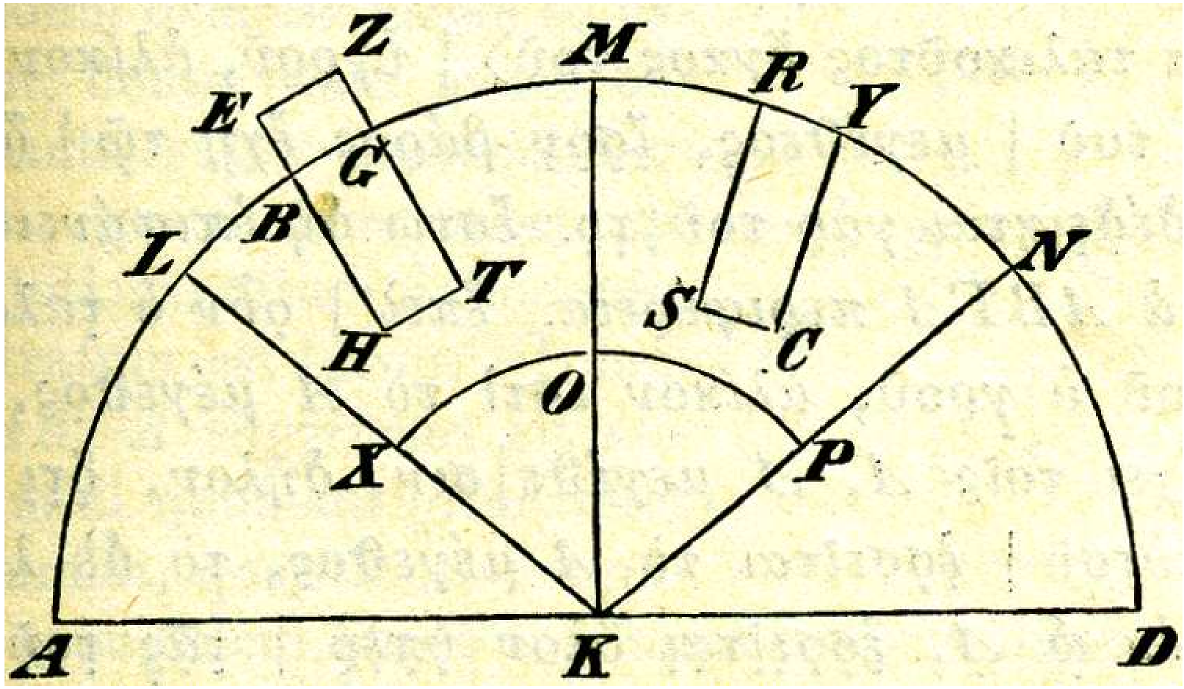


5. Shrinking and rotating.



... and at the end the most famous:

DE CORPORIBUS FLUITANTIBUS.



Opera ed. Heiberg 1910, p.89, BGE FL10626

Proposition V: Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced.

Not astonishing that Leibniz wrote ...

“Qui Archimedem et Apollonium intelligit, recentiorum summorum virorum inventa parcius mirabitur.”

[Those who perceive the works of Archimedes and Apollonius will marvel less at the discoveries of the greatest modern scholars.]

(G.W. Leibniz; copied from Ver Eecke 1923)

(P.S. This was perhaps also a little pin-brick against the glory of Newton.)