The Birth of Astronomy (and Science)

Brissago, 27. Sep. 2011

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Kepler

Newton

Gauss

(Gravity ...) was one of the first great laws to be discovered and it has an interesting history. You may say, ‘Yes, but then it is old hat, I would like to hear something about a more modern science’. More recent perhaps, but not more modern. (...) I do not feel at all bad about telling you about the Law of Gravitation, because in describing its history and methods, the character of its discovery, its quality, I am being completely modern.

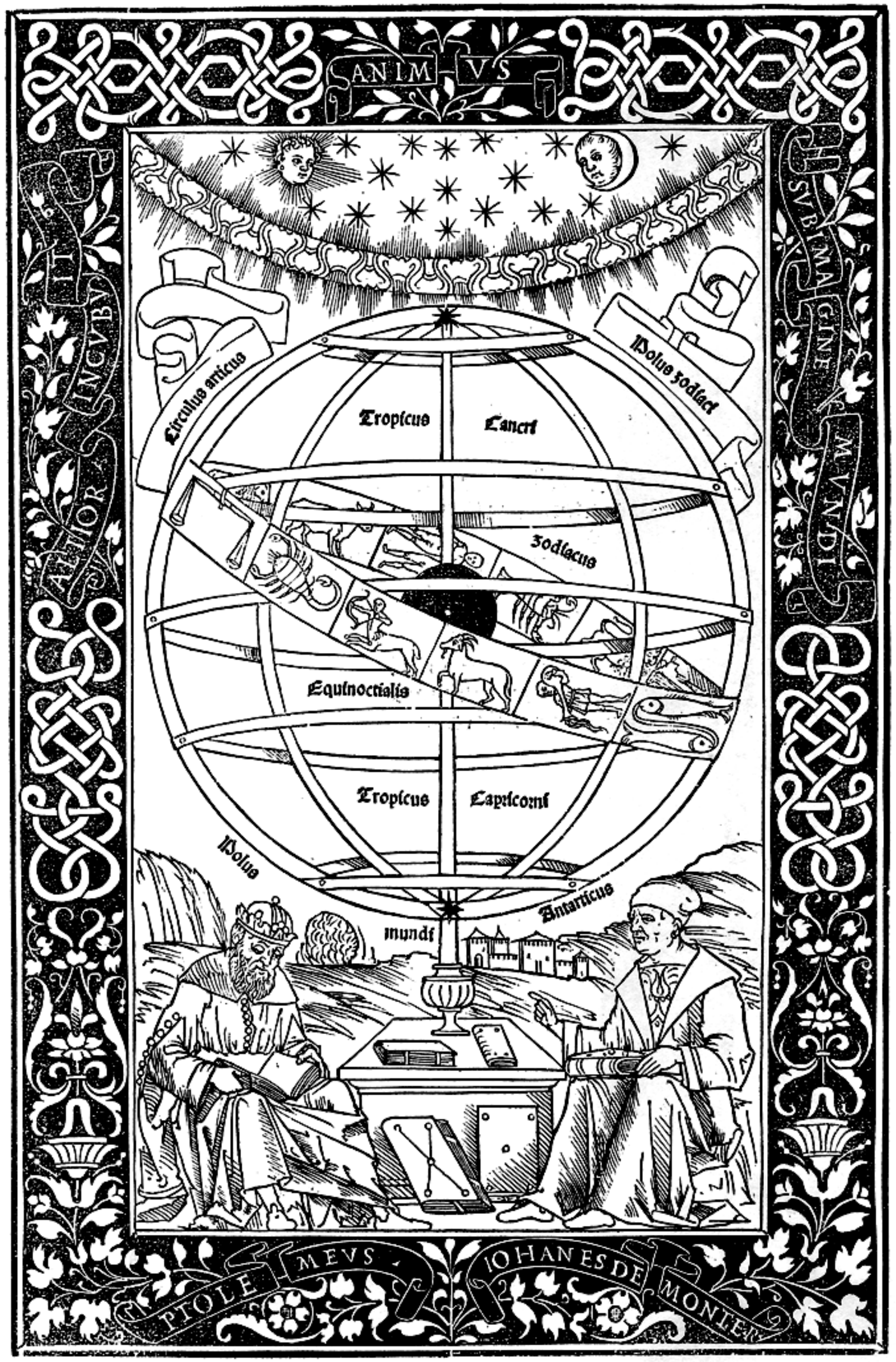
(R. Feynman 1965)

Astronomy is older than physics. In fact, it got physics started by showing the beautiful simplicity of the motion of the stars and planets, the understanding of which was the beginning of physics.

(R. Feynman, published in *Six easy pieces*, 1963, p. 59.)

$\Rightarrow$ ... AND Science ... !!

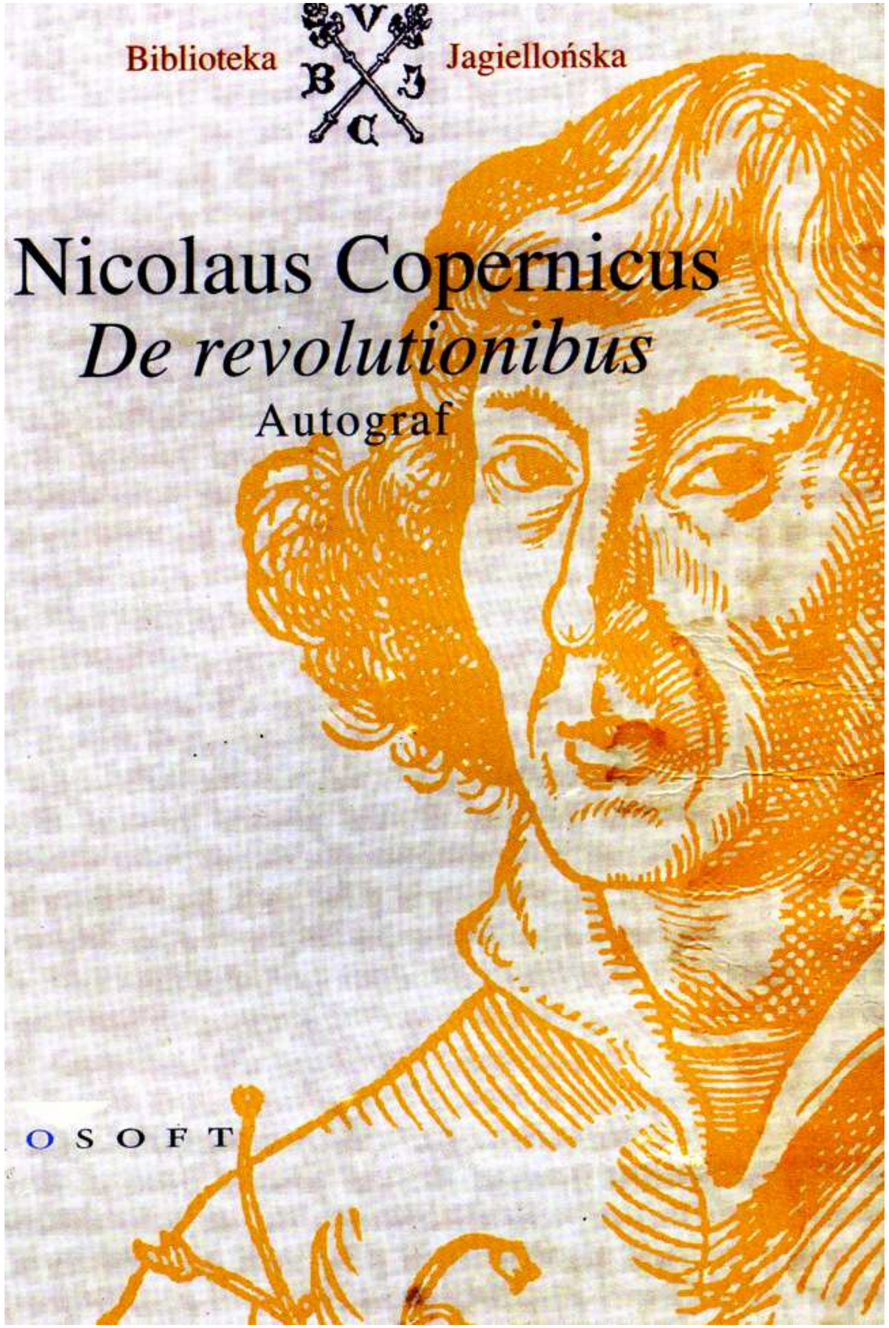
The Birth of Science: Ptolemy 150 A.D. “Almagest”

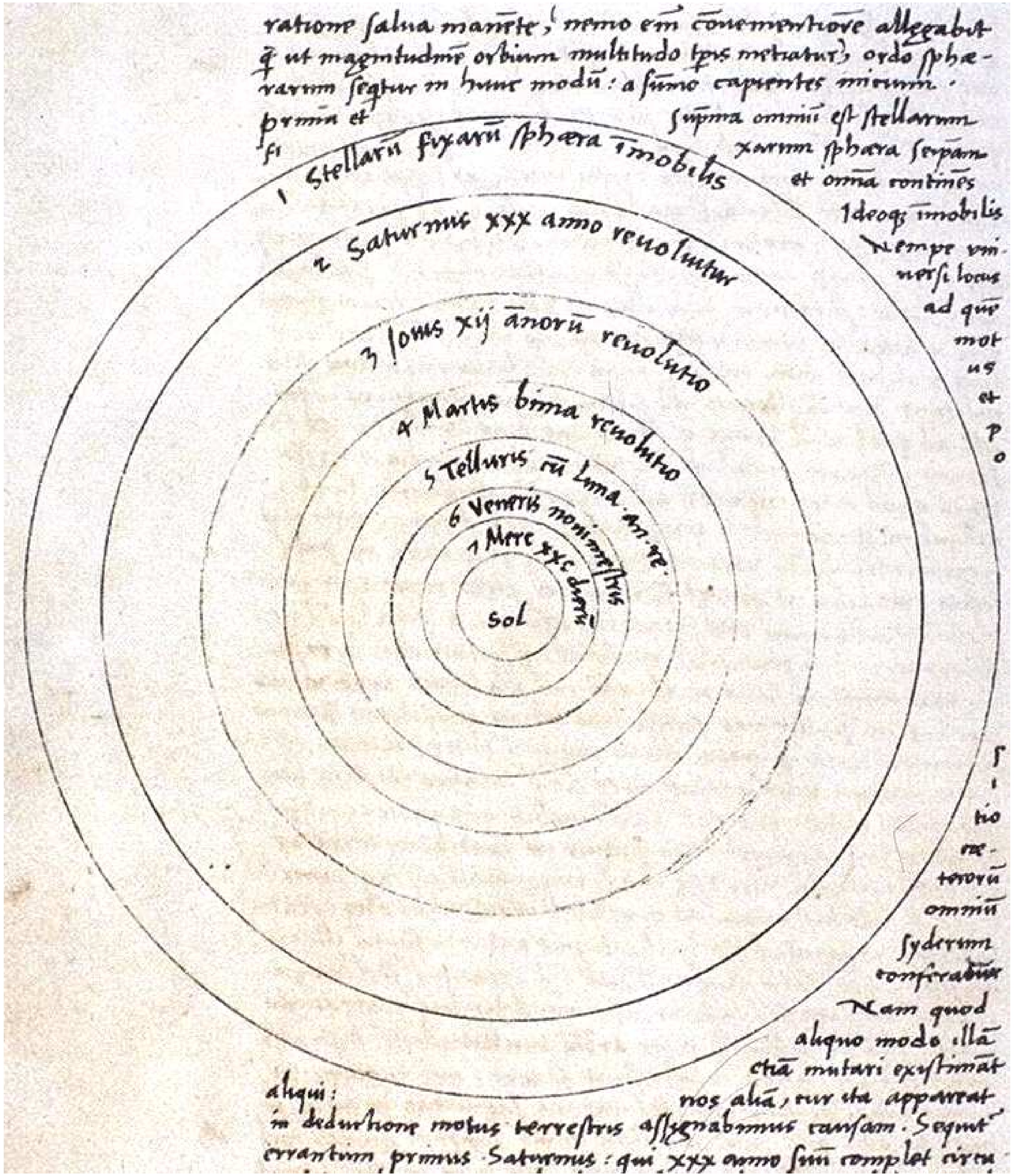


Cl. Ptolemei alexandrini Astronomorum principis ισ μεγαλὶν σὶντα ξὶν id est in Magnam Con/structionē: Georgij purbachij: eiusque di/scipuli Johannis de Regio monte Astronomicon Epitoma.

(publ. 1496 by Regiomontanus)

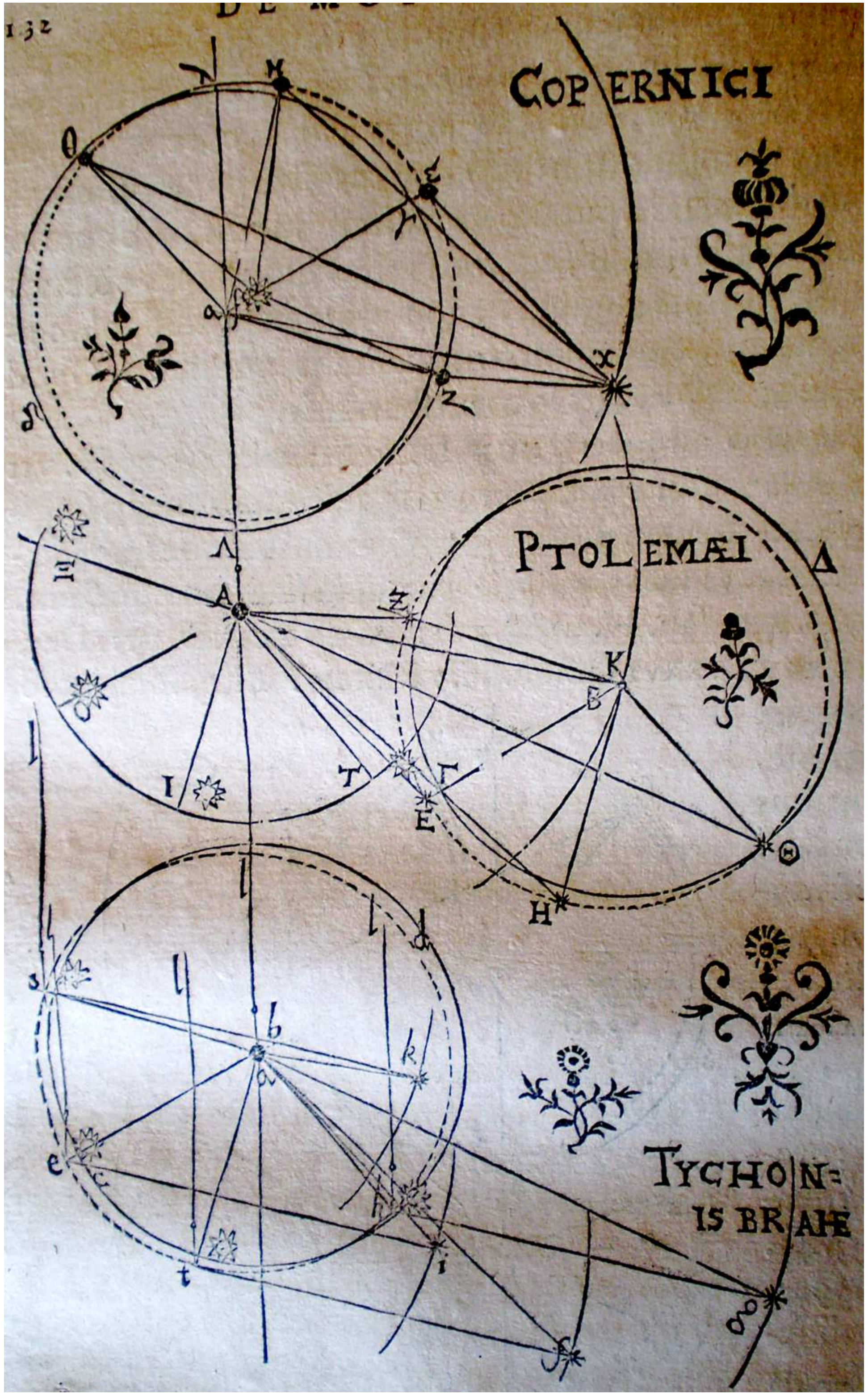
Copernicus (publ. 1543)





Tycho Brahe (1546 – 1601):

“I start by explaining these things in the Copernican setting, where they are easiest to understand.”
(Kepler 1609,
(chap. XXIV, p. 131).

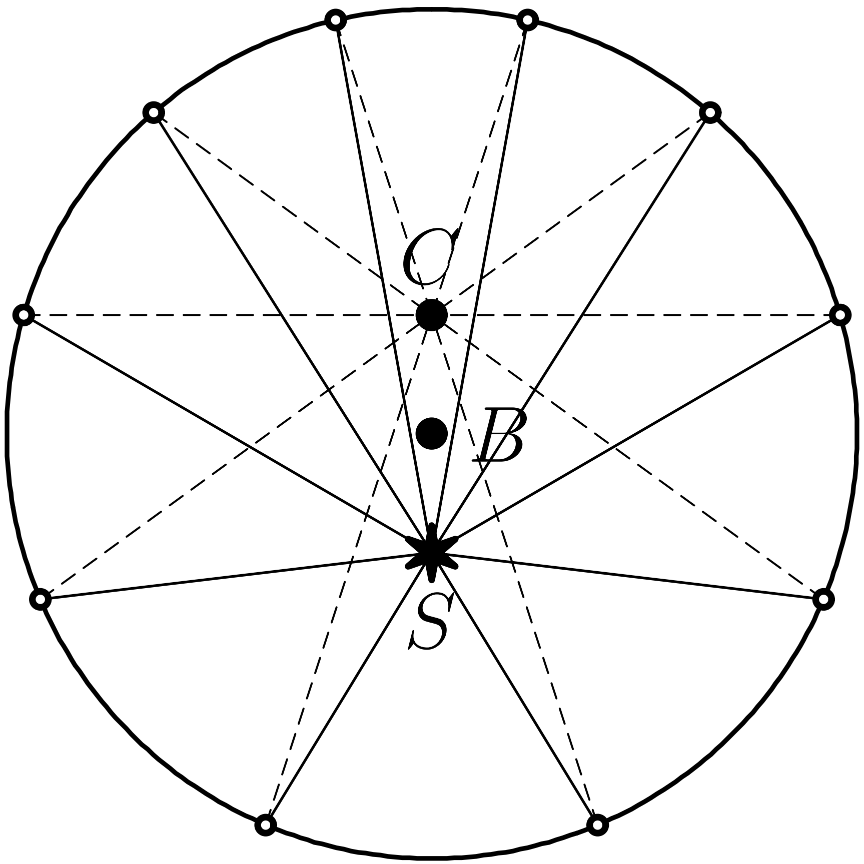


Ptolemy – Copernicus – Brahe : same for relative movement:

$\Rightarrow$ orbits are excentric circles !!
(Inaequalitatis primae)

rotation speed governed by
"punctum aequans" C
with $CB = BS$
(Inaequalitatis secundae)

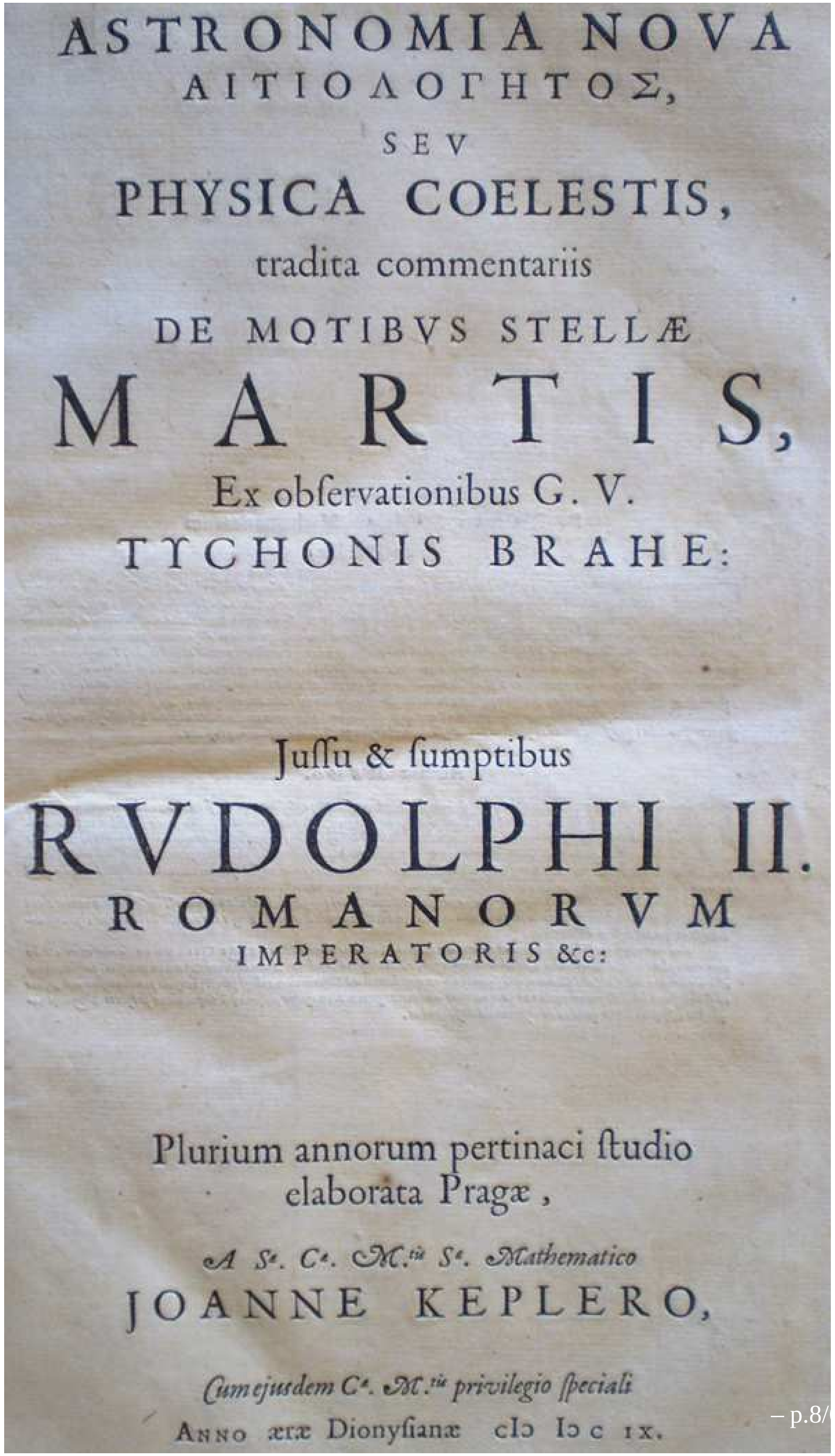
Thousands of data (Brahe)
of unprecedented precision
to adapt parameters



S: Sun,
B: "Mean" Sun,
C: punctum aequans.

Worked fine for all planets **except Mars !!**

Johannes Kepler (1609):
Great Triumph:



ASTRONOMIA NOVA
ΑΙΤΙΟΛΟΓΗΤΟΣ,
SEV
PHYSICA COELESTIS,
tradita commentariis
DE MOTIBVS STELLÆ
MARTIS,
Ex obſervationibus G. V.
TYCHONIS BRAHE:

Juſſu & ſumptibus
RVDOLPHI II.
ROMANORVM
IMPERATORIS &c:

Plurium annorum pertinaci ſtudio
elaborata Pragæ,

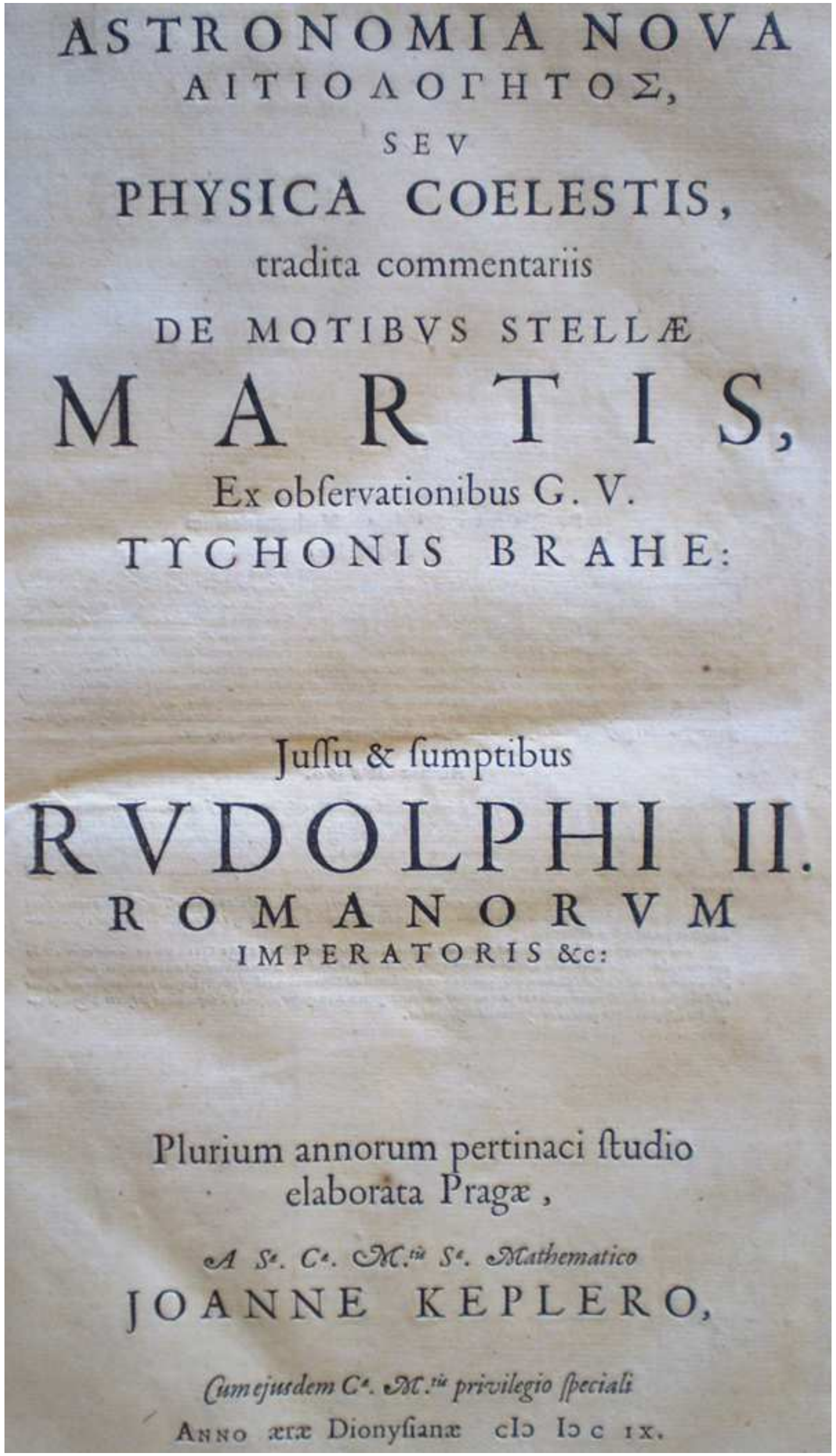
A S. C. M.tis S. Mathematico
JOANNE KEPLERO,

Cum ejusdem C. M.tis privilegio ſpeciali
ANNO æræ Dionyſianæ cIↄ Iↄ c ix.

Stets zweifelte er (Kepler) an sich und seiner Arbeit. Seinen eigenen Vortragsstil empfand er als "abstossend oder jedenfalls verwickelt und schwer verständlich". Über eines seiner Hauptwerke, die 1609 erschienene "Astronomia Nova", urteilte er im Nachhinein: "Ich selber, der ich als Mathematiker gelte, ermüde beim Wiederlesen meines Werkes mit den Kräften meines Gehirns."

(from Spiegel online, 26.07.2011)

SYNOPSIS TOTIUS OPERIS :



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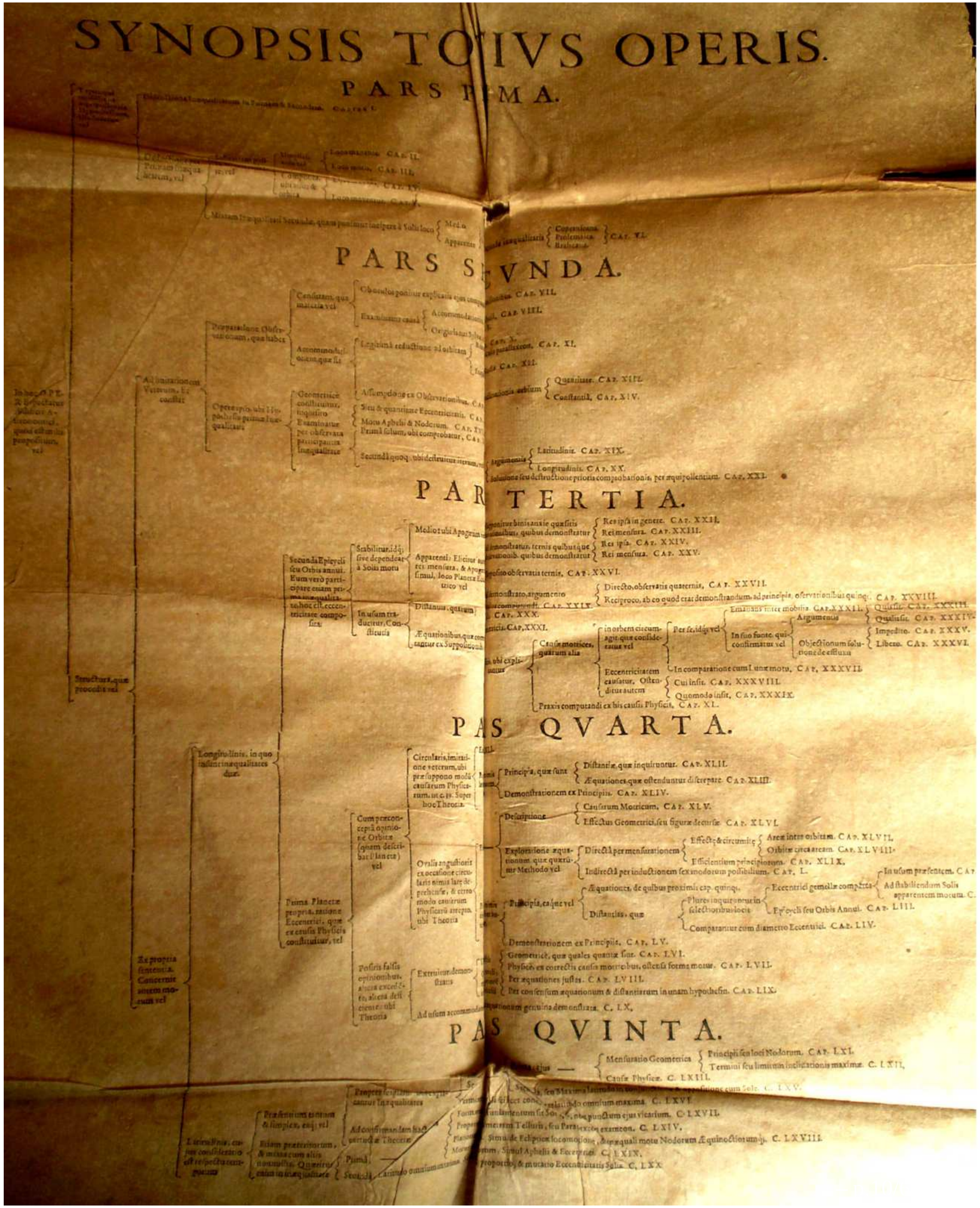
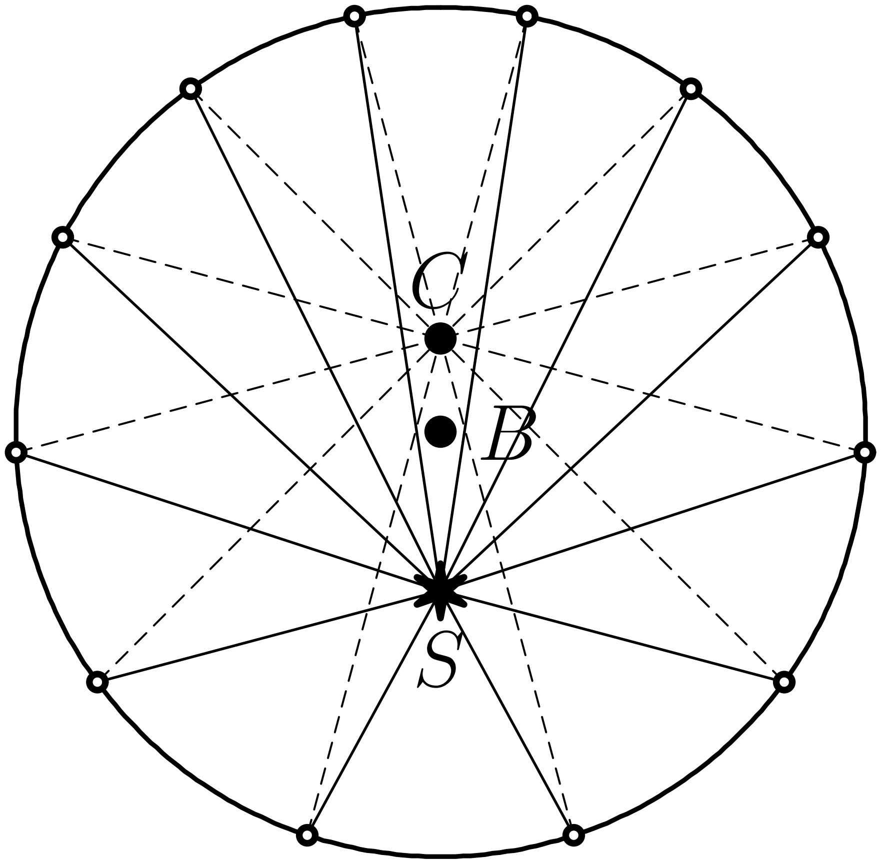
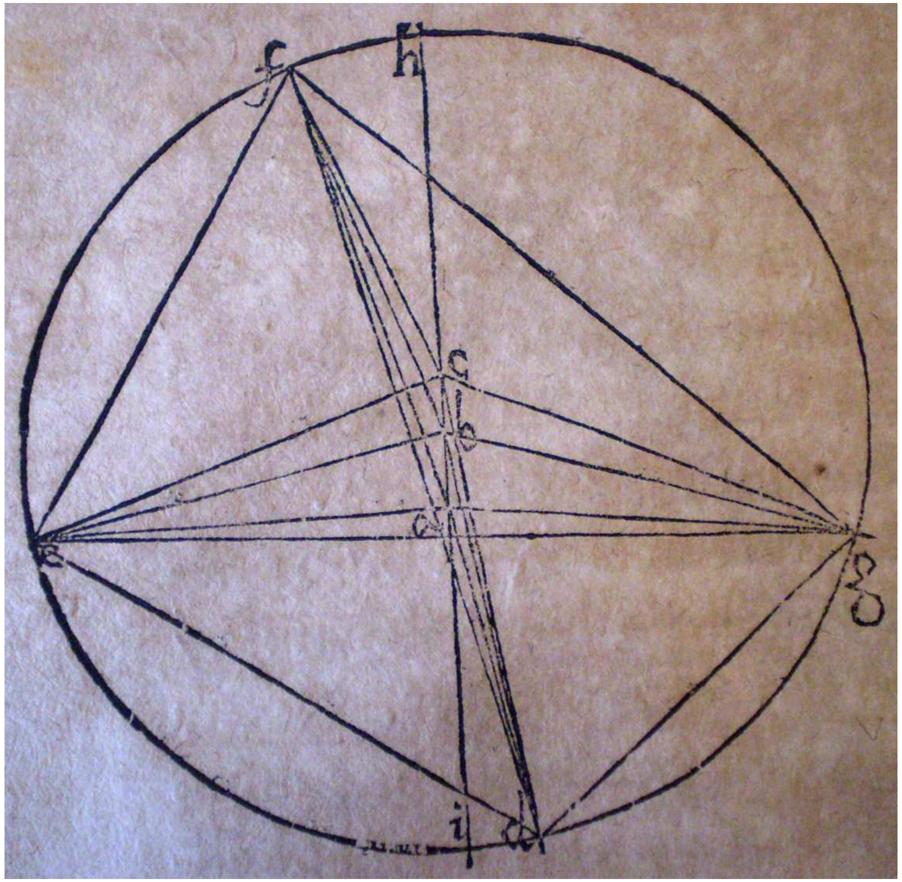


SYNOPSIS TOTIVS OPERIS.
PARS PRIMA.
PARS SECVNDA.
PARS TERTIA.
PARS QVARTA.
PARS QVINTA.

Kepler's Pars Secunda : "Ad imitationem veterum"

(the "Ancients" are Ptolemy, Copernicus and Tycho Brahe, 25 years older than he).



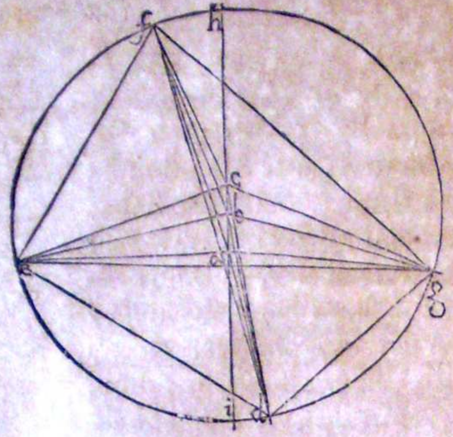


No justification for Ptolemy's assumption $CB = BS$;
$\Rightarrow$ determine S, B and C
from 4 observations (solve numer. 4-dim. nonl. system)

"If you find these calculations
disgusting (pertaesum),
then have pity for me
(jure mei te miserat),
I did them at least 70 times
losing a lot of time
(ad minimum septuagies ivi
cum plurima temporis jactura)".

$\Rightarrow$ already good
approximations for Earth's orbit
Hypothesis vicaria !!

(for details see S. Thorvaldsen 2010)



CAPVT XVIII.

Examen duodecim locorum acronychiorum per inventam hypothesin.

Vtar autem ea calculi forma, quam supra cap. IV explicavi quod sit compendiosior. Certum autem est in COPERNICANA seu TYCHONICA forma non sesquiscrupulum (imo minus aliquid) vel lucratum vel perditũ iri, ut ibidem monui.

	Anno 1580	Anno 1582	Anno 1585	Anno 1587	Anno 1589
Aphel. anno 1587	28.48.55. ♌	4.28.48.55.	4.28.48.55.	4.28.48.55.	4.28.48.55.
Movetur annis intermediis	6.42.	4.28	2.14	0.	2.15.
Aphel. anno suprascripto	4. 28.42.13.	4.28.44.27.	4.28.46.41.	4.28.48.55.	4.28.51.10.
Longitudo media	1. 25.49.31.	3. 9.24.55.	4.20. 8.19.	6. 0.47.40.	7.14.18.26.
Adde	3.55.	3.55.	3.55.	3.55.	3.55.
Correcta long. med.	1. 25.53.26.	3. 9.28.50.	4.20.12.14.	6. 0.51.35.	7.14.22.21.
Ergo angulus C	87.11.13.	49.18.37.	8.14.27.	32. 2.40.	75.31.11.
Sinus	99880	75767	7232	7232	
Eccentricitas aequantis.	7232	7232	14909	53058	96833
	65088	50624	07232	36160	65088
	6509	3616	2893	2169	4339
	579	506	651	36	578
	58	43	6	6	14
		5			2
	7223	5479	1078	3837	
Pars aequation.	4. 8.33.	3. 8.26.	0.37. 4.	2.11.57.	
	91.19.46.				7002
Angulus B	88.40.14.	46. 7.11.	7.57.23.	29.50.43.	4. 0.55.
Dimid.	44.20. 7.	23. 3.36.	3.58.42.	14.55.21.	71.30.16.
Tangent.	97706	79643	79643	79643	35.45. 8.
Quotiens qui prodit ex divisione differentiae laterum in Summam	79643	42572	6955	26650	72002
	716787	318572	47786	159286	557501
	58750	15929	7168	47786	15929
		3982			
	5575	507	398	4779	06
	48	16	40	398	
Tangent.	778160	33906	5539	21225	57349
	37.53.22.	18.43.47.	3.10.13.	11.59. 0.	29.49.54.
	44.20. 7.	23. 3.36.	3.58.42	14.55.21.	[illegible]45. 8.

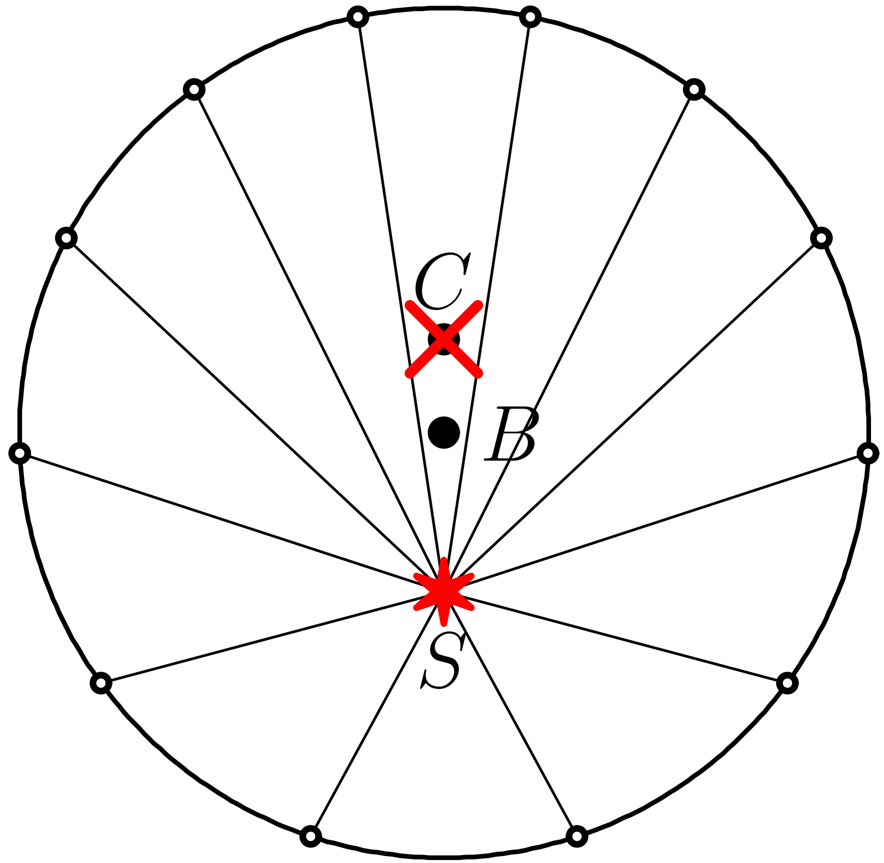
Pars Tertia : “Ex propria sententia” (his own opinion)

Kepler, as convinced Copernician,
puts **Sun** in the center;
get rid of C,
which has no physical meaning:

ΑΙΤΙΟΛΟΓΗΤΟΣ

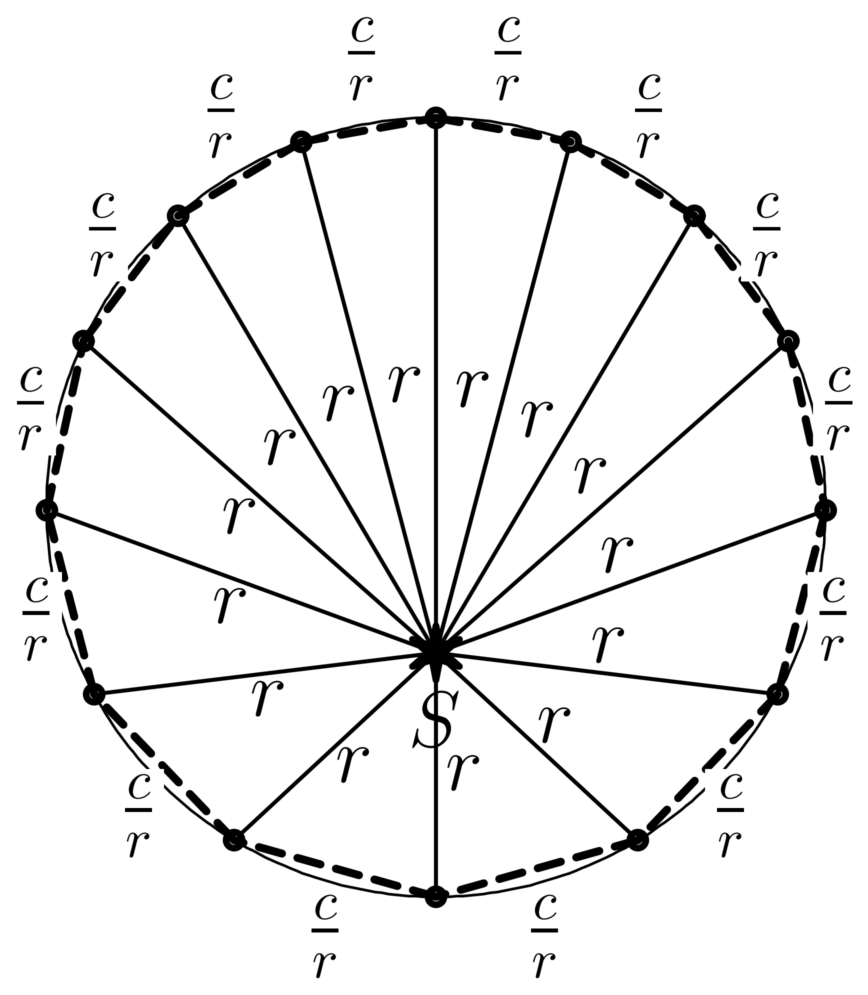
PHYSICA COELESTIS,

Which theory for rotation speed ??



Pars Tertia : "Ex propria sententia" (his own opinion)

Long discussions,
(Chap. 32–39)
attractive forces, magnetism,
rotating Sun "pushes" the planets,
the planets have a "Soul" ;
the planets "wish" to move;
the planets "look" at the Sun and
see diameter inv. prop. to r
⇒ Speed inversely prop. to r !!
(New Inaequalitatis secundae).



Chap. 40: End of Pars Tertia : Simplified model:

C A P V T XL.

Methodus imperfecta æquationes ex Phyſica hypotheſi computandi, quæ tamen ſufficit theoriæ SOLIS vel TERRÆ.

Above model too complicated ...

⇒ Inspired by Archimedes

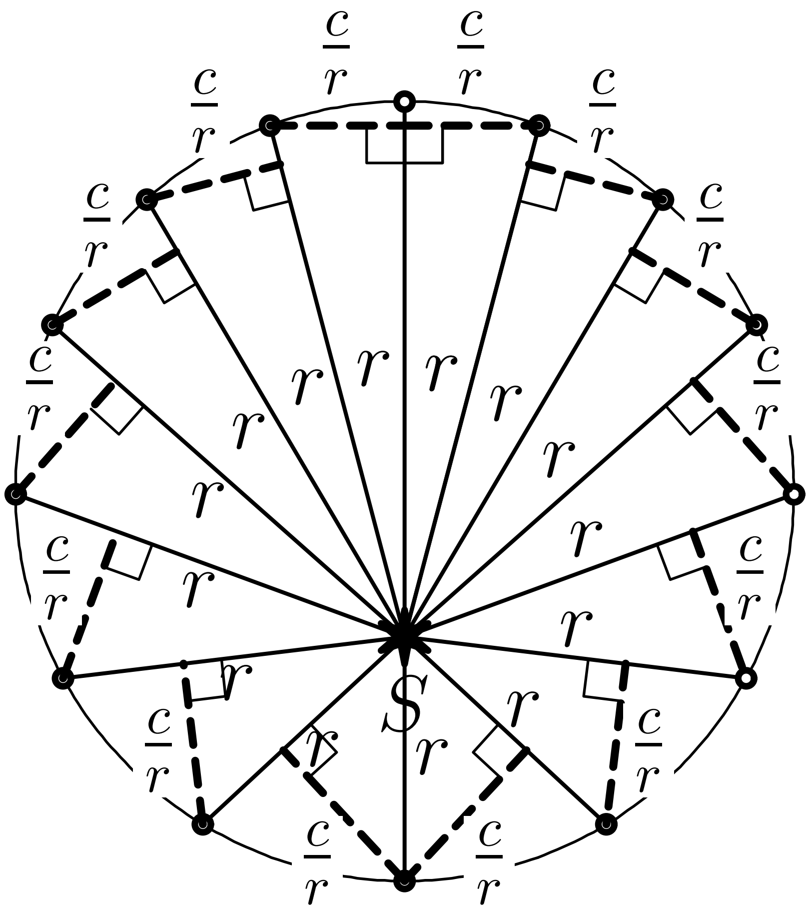
diſtantias omnes ineſſe. Nam memineram, ſic olim & ARCHIMEDEM, cum circumferentiæ proportionem ad diametrum quæreret, circulum in infinita triangula diſſecuiſſe. nam hæc vis occulta eſt ejus demon-

⇒ Replace hypothenuse by leg

⇒ all triangles have same area !!

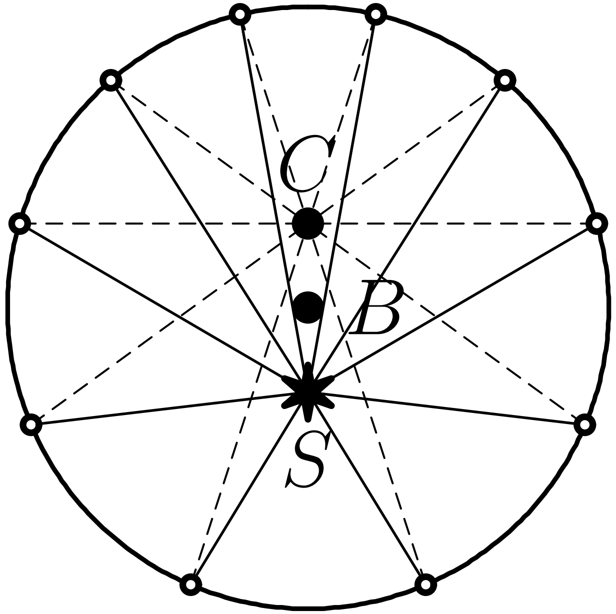
"Equal times — equal areas"

(Correct Inaequalitatis secundae).

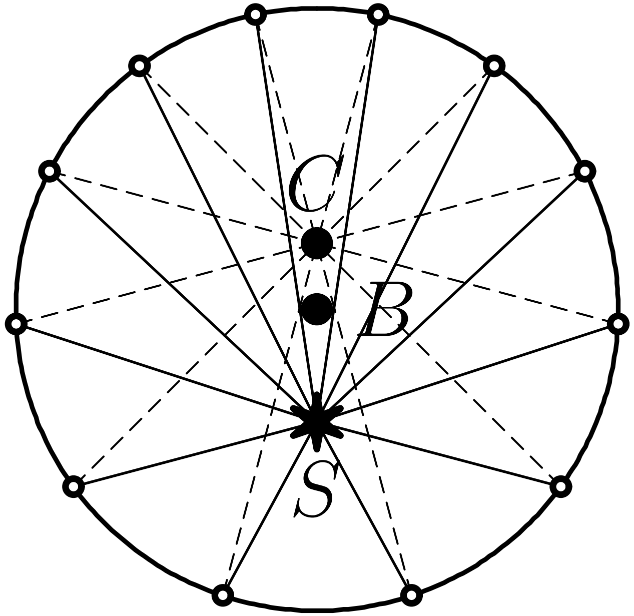


Resumé:

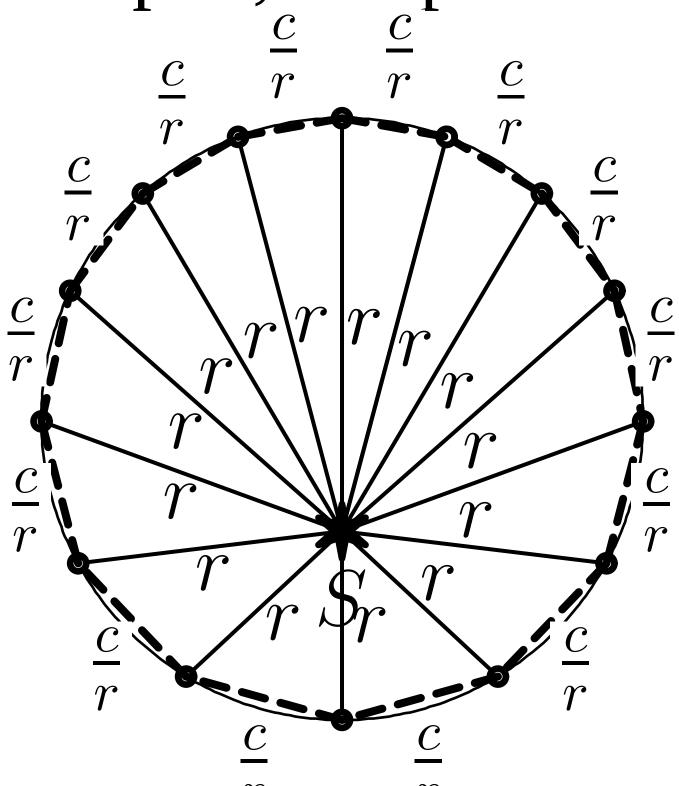
Ptolemy, Cop., Brahe:



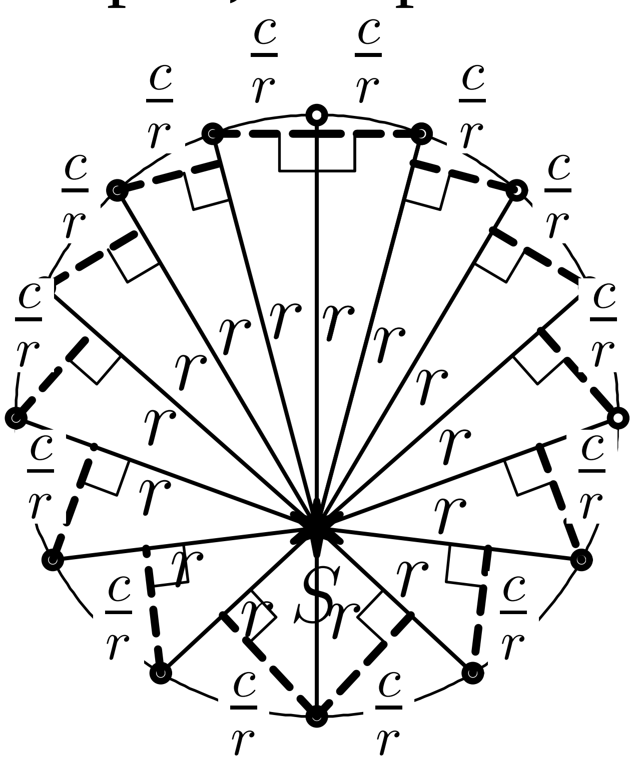
Kepler, Hypoth. vicaria:



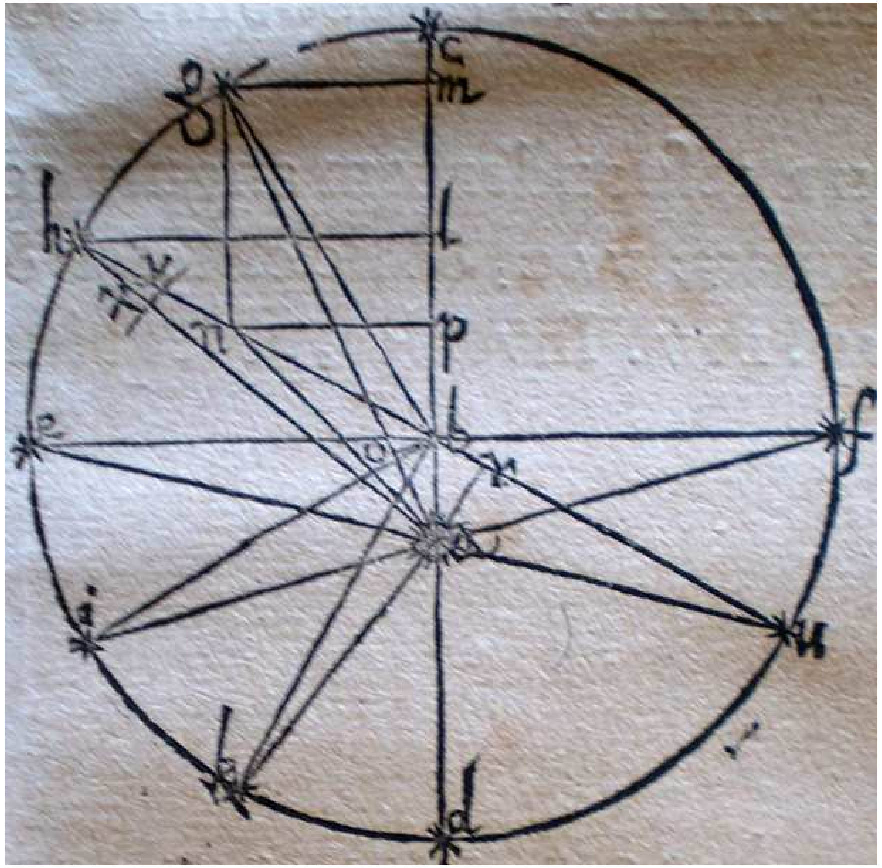
Kepler, Chap. 32-39:

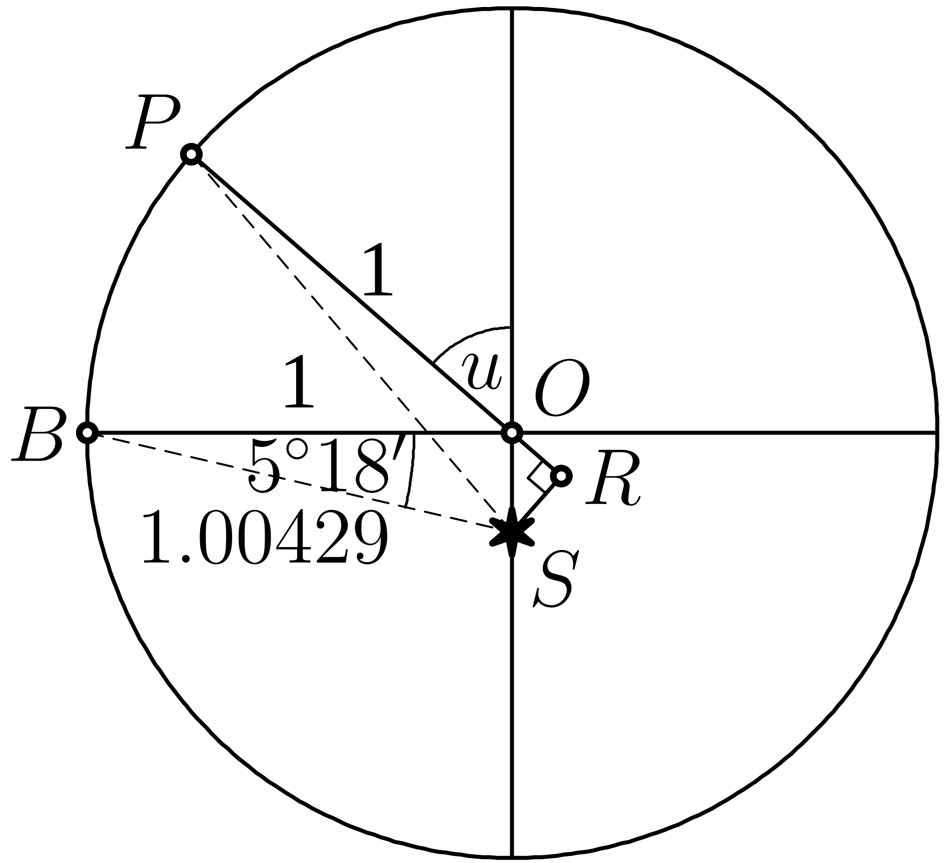


Kepler, Chap. 40:



Kepler’s Pars IV : The Great Idea in Chap. 56:

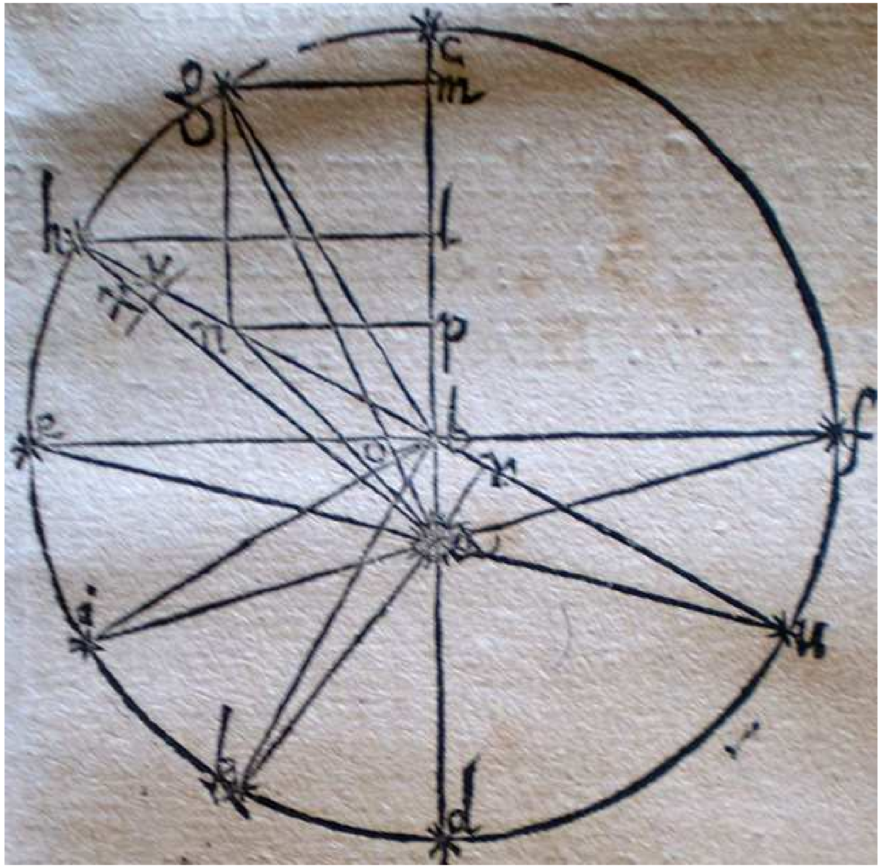


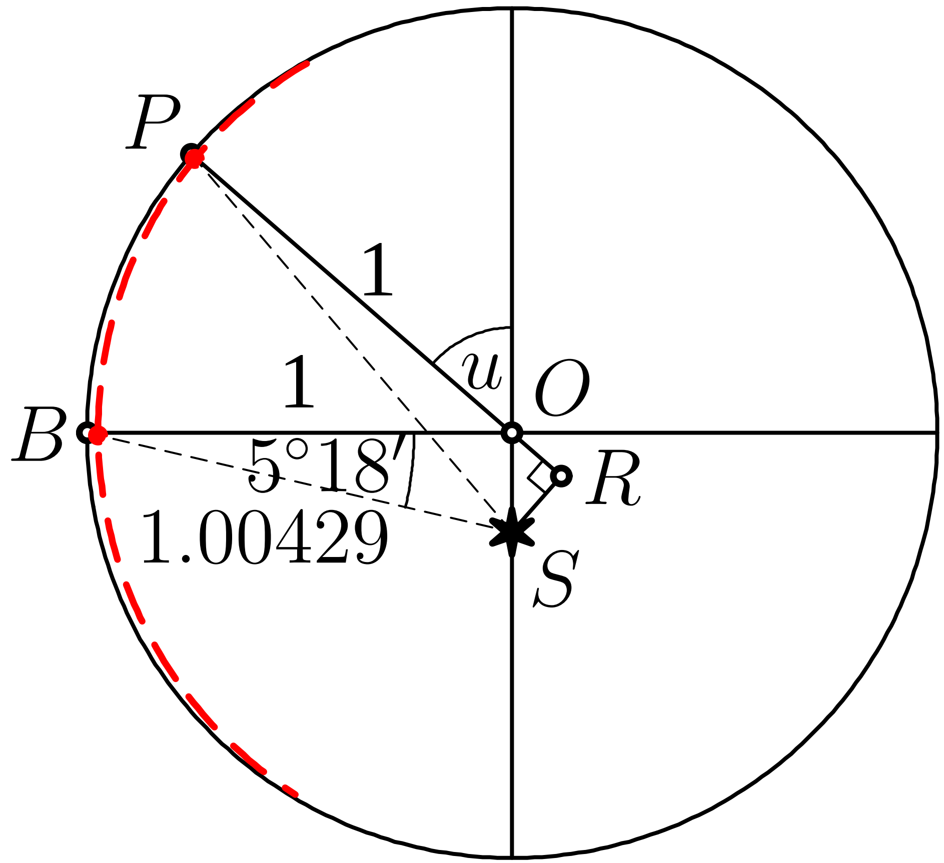


Obs.: Dist. BS of Tycho’s circle by factor 1.00429 too large;
This value is (by chance) $1/\cos 5°18'$;

plane nihil dictum eſſe, itaque futilem fuiſſe meum de Marte trium-
phum; forte fortuito incido in ſecantem anguli 5°. 18′. quæ eſt menſura
æquationis Opticæ maximæ. Quem cum viderem eſſe 100429, hic
quaſi e ſomno expergefactus, & novam lucem intuitus, ſic cœpi ratio-

Kepler's Pars IV : The Great Idea in Chap. 56:



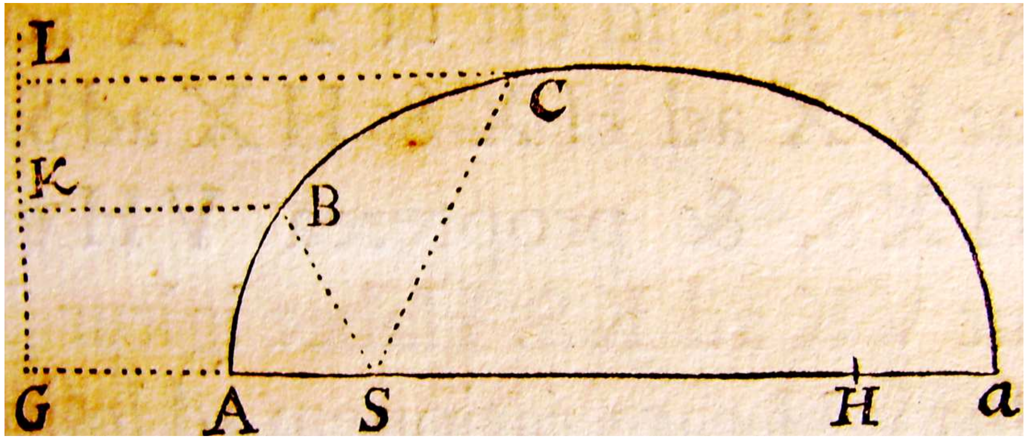


Idea: Replace 'hypoth.' by 'legs', $BS = BO$, $PS = PR$,...
and "I awoke from sleep & new light broke on me"!!!

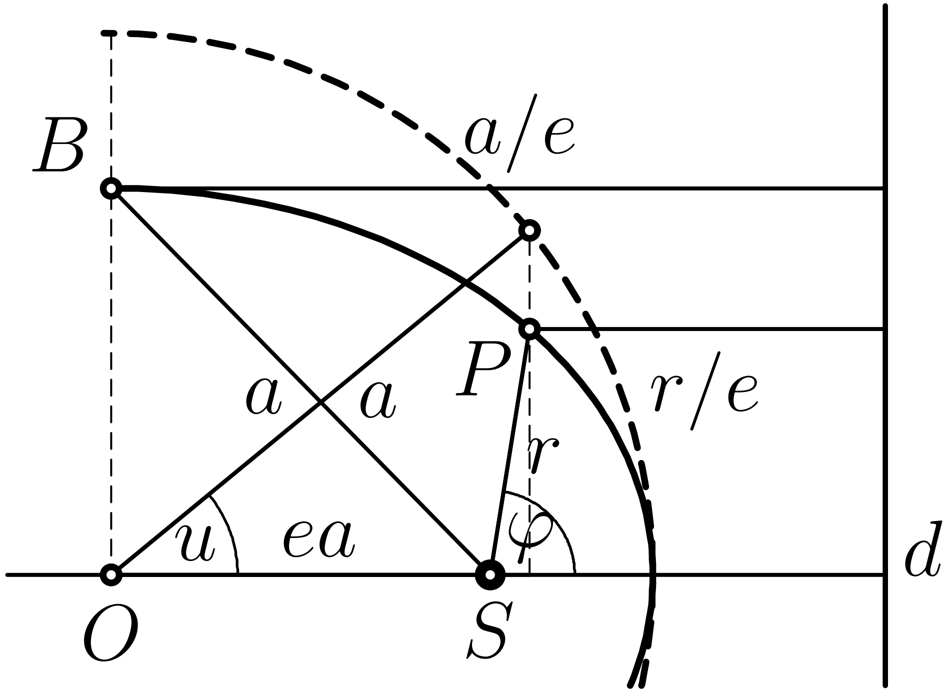
$$PS = PR = 1 + e\cos u$$

Hoc jam obtento, non rationibus a priori, sed observationibus,... (This is now established, not from reasoning as before, but from observations...)

Pappus of Alexandria $\approx$ 300 A.D. *Collection*, Prop. VII.238



(drawing Newton 1687)



$$a\cos u + \frac{r}{e} = \frac{a}{e} \quad \Rightarrow \quad \boxed{\begin{array}{l} r = a - ea\cos u \\ r = a + ea\cos u \end{array}}$$

Kepler:

$PS = PR = 1 + e\cos u$ (Ch. 57–59) $\Rightarrow$ **orbit is ellipse !!**

EXISTENT acuti Geometræ VIETÆ ſimiles, qui magnum aliquid
eſſe putabunt demonſtrare hujus METHODI ἀτεχνίαν. Id enim & PTO-
LEMÆO & COPERNICO & REGIOMONTANO objectum in hoc negocio
a VIETA. Eant igitur & ſchema Geometrice ipſi ſolvant, & erunt
mihi magni Apollines. MIHI ſufficit ad quatuor vel quinque con-

“There might be ingenious Geometers, similar to Viète, who critizise with great emphasis this [numerical] METHOD as being *artless*. Indeed, Viète critizised in such a way Ptolemy, Copericus & Regiomontanus in his work. May they step forward then, and solve themselves the scheme Geometrically, they will be great Apollons for me.”

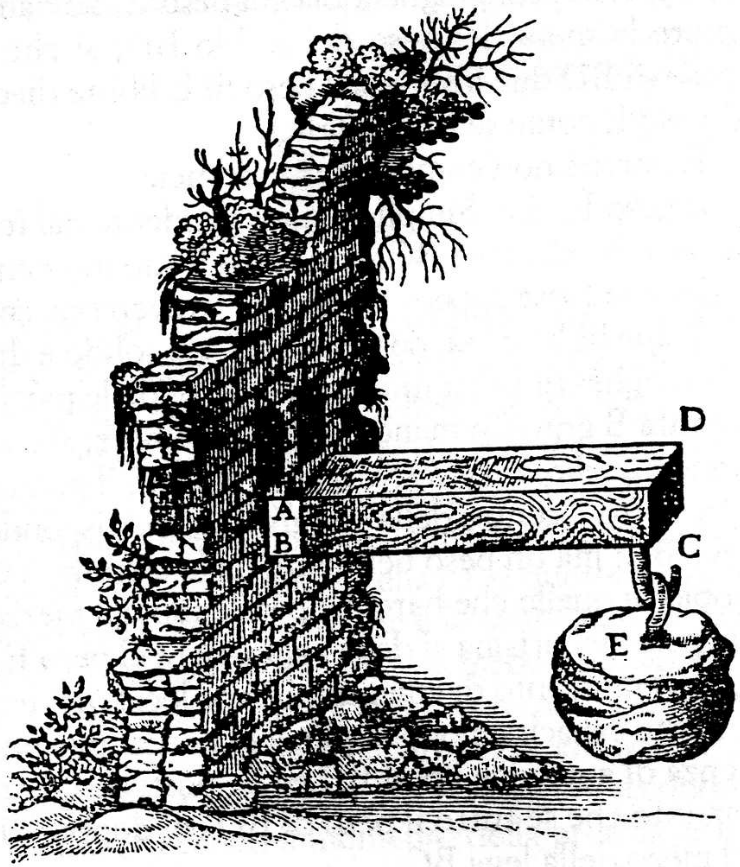
(Kepler 1609, p. 95).

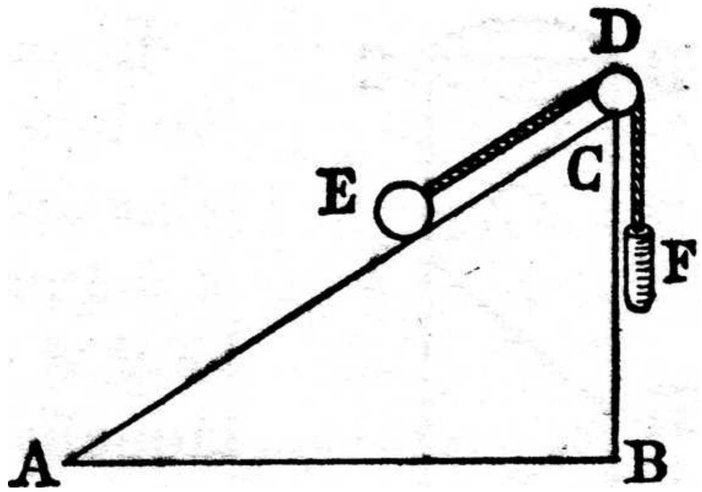
Galileo Galilei. Mechaniques 1634, Discorsi 1638:

... io grandemente dubito che Aristotele non sperimentasse ...

(Galilei 1638, Giornata prima)

Observation of Heaven...





...study forces, inertia and accelerated motion on Earth

“The Great Apollon” : Isaac Newton.

... one of the most dramatic moments of the *real beginnings* was when Newton suddenly understood *so* much from *so* little... (R. Feynman, lecture of march 13, 1964.)

This sudden change of emphasis has been provoked by a visit from Edmund Halley (1656–1742), which probably took place in August [1684].” (quoted from *Footprints of the lion* by Scott Mandelbrote, Cambridge 2001, p. 88)

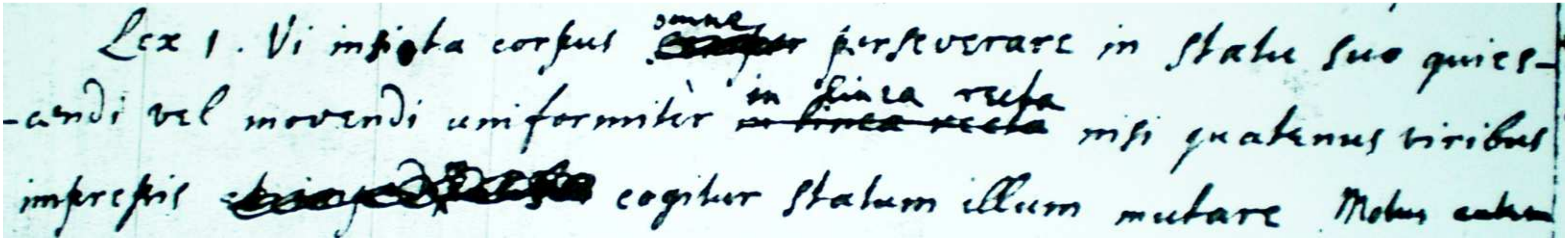
Manuscript Add 3965^{7a}
from 1684
written in **this room** $\Rightarrow$ $\Rightarrow$
leading to the
Principia published 1687

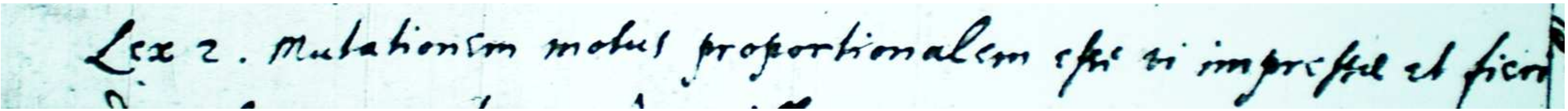
Proof of Kepler 2:

... Newton was known in Cambridge as 'the man who has writt a book that neither he nor any one else understands' ...

(quoted from *Footprints of the lion* by Scott Mandelbrote, Cambridge 2001, p. 28)

Let **us** try to understand:


Lex 1. Vi insita corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in linea recta nisi quatenus viribus impressis cogitur statum illum mutare. Motus autem

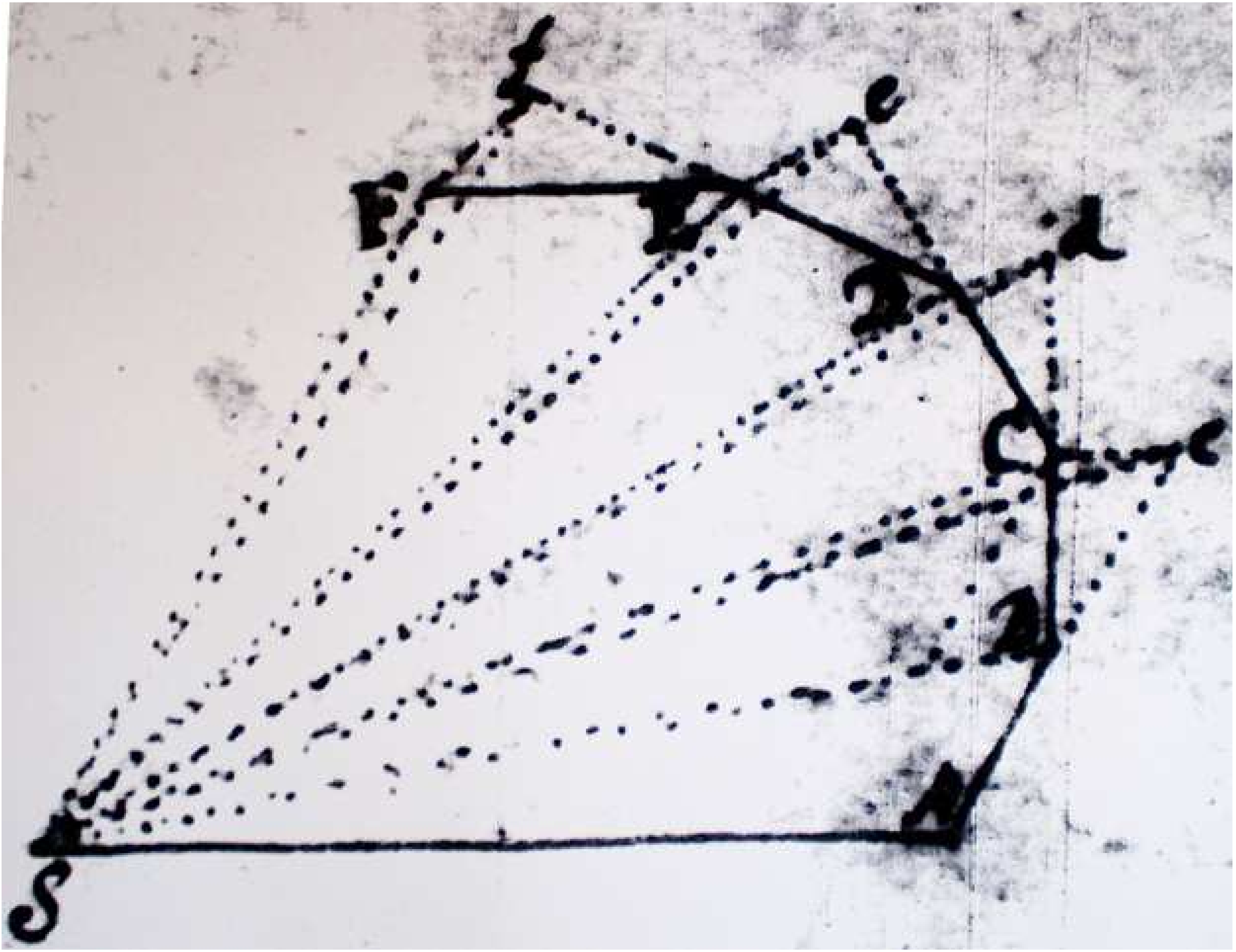

Lex 2. Mutationem motus proportionalem esse vi impressae et fieri

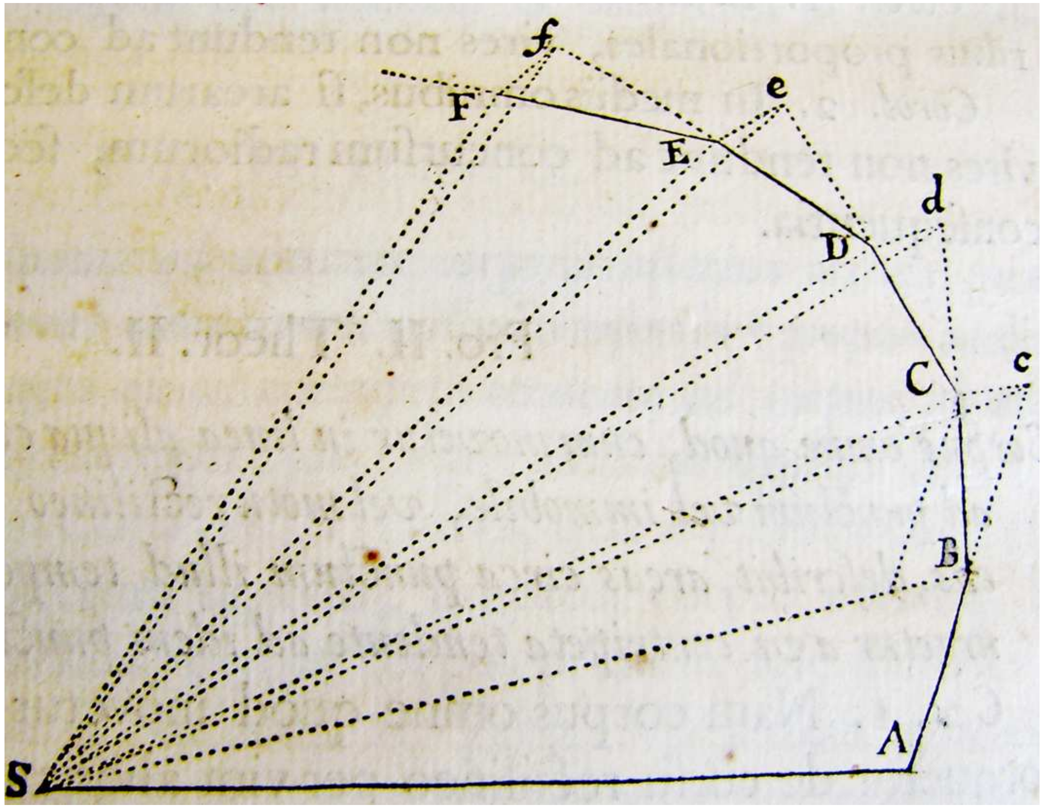
(Newton, manuscript Add. 3965^{7a})

Idea:

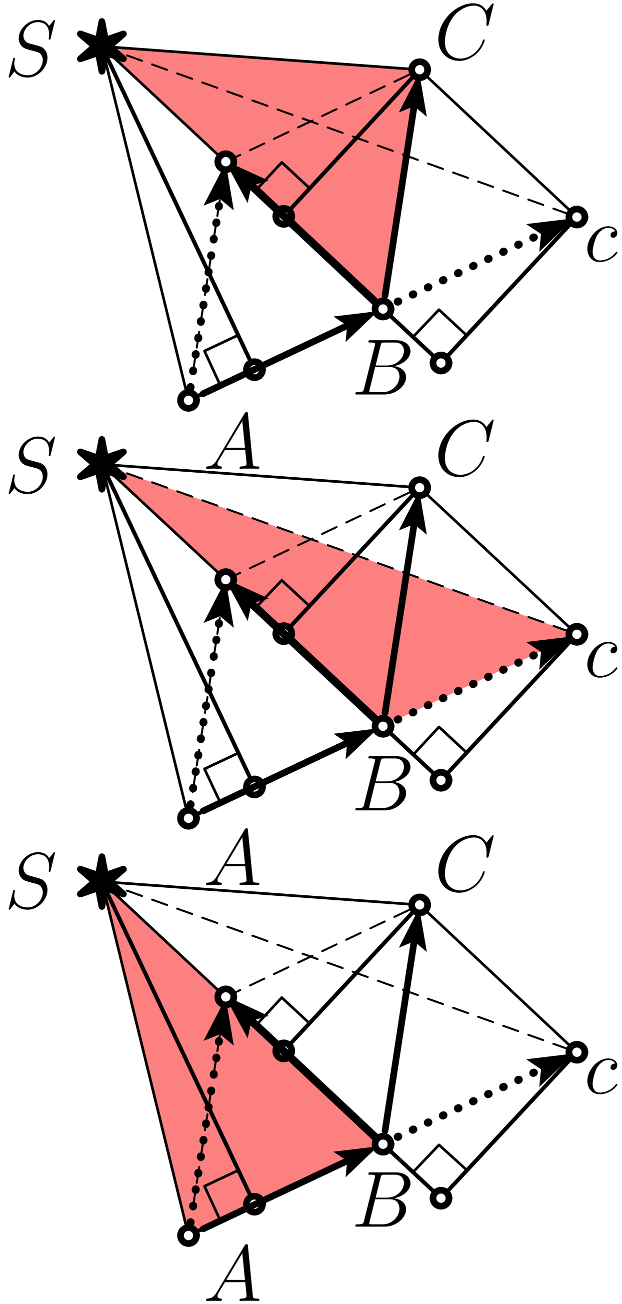
Instead of continuously acting force

$\Rightarrow$ force impulses $f \cdot \Delta t$ at end of time steps





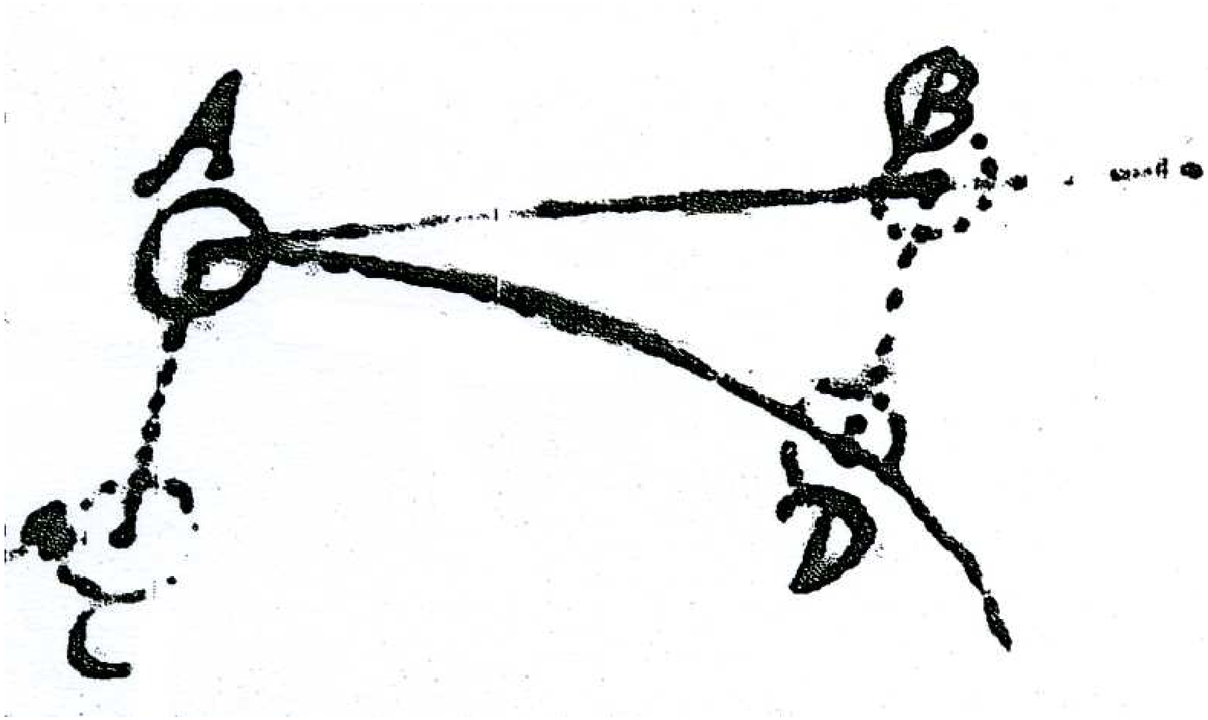
Picture from *Principia*



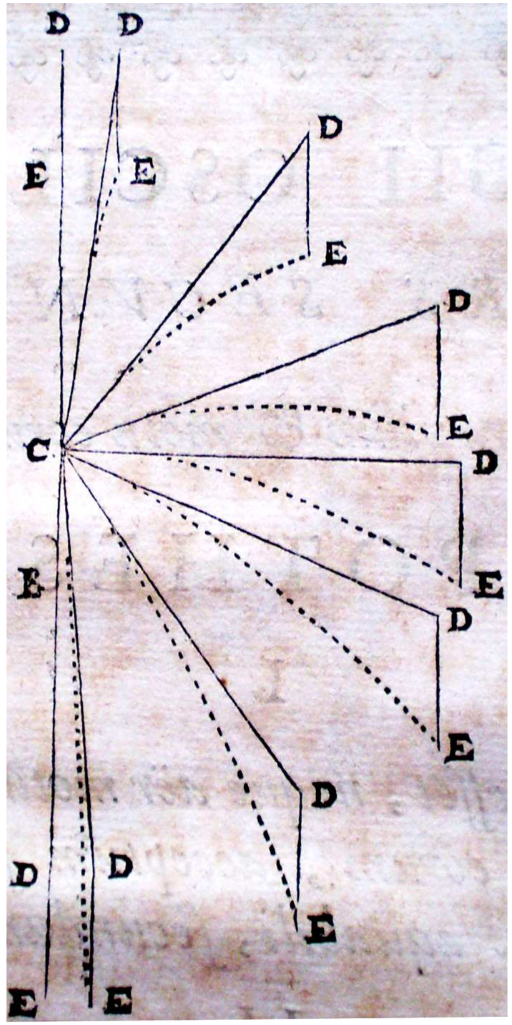
Eucl. I.41: All triangles have same area !

This became the "Theorema 1" of the *Principia* (1687).

Newton's Discovery of Gravitation Law from Kepler 1 & 2.



(Newton, Add 3965^6, 1684)



(Huygens 1673)

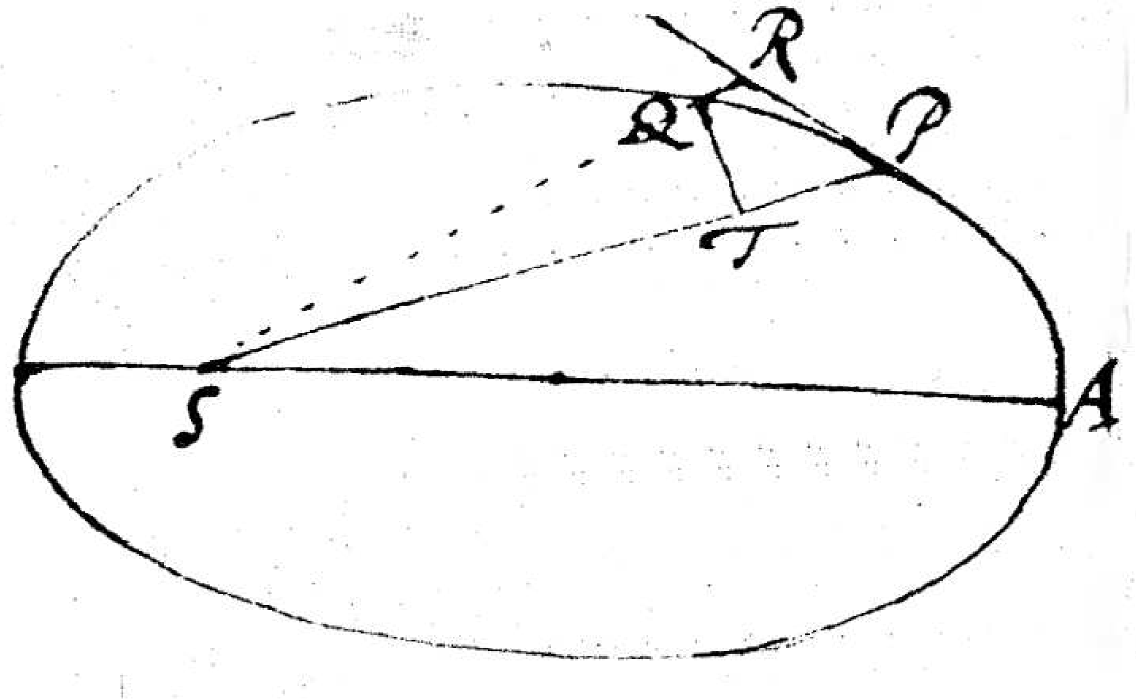
Thm.: Force impulse prop. dist. R on tangent and Q on orbit (Δt fixed).

Newton's "Prob. 3".

If orbit ellipse
and S in focus,
then, for small step-sizes,

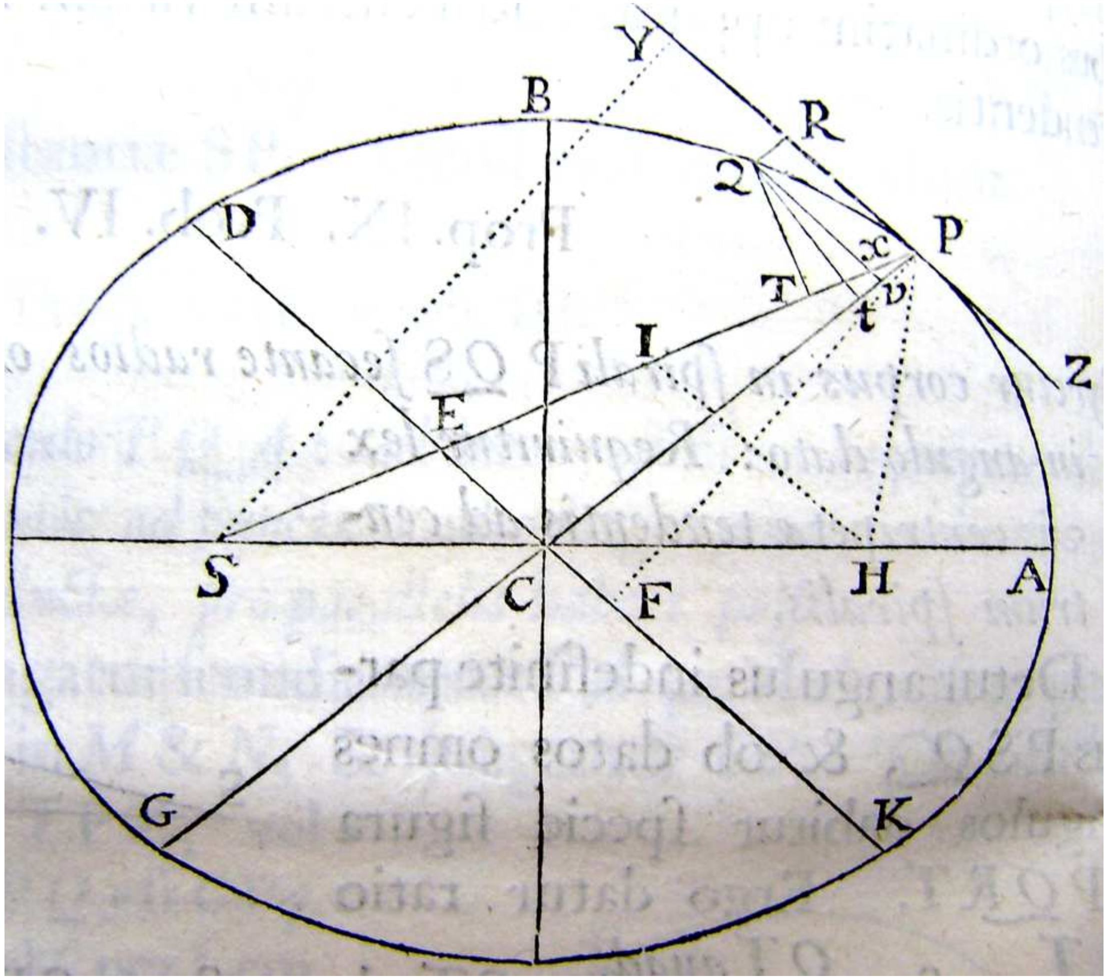
$RQ \approx \text{Const} \cdot QT^2$

where Const independent
of position of P.



Proof: Apollonius of Pergae $\approx$ 250 B.C.

... i libri di Apollonio, ... delle quali sole siamo bisogni nel presente trattato. (Galilei 1638, giornata quarta)



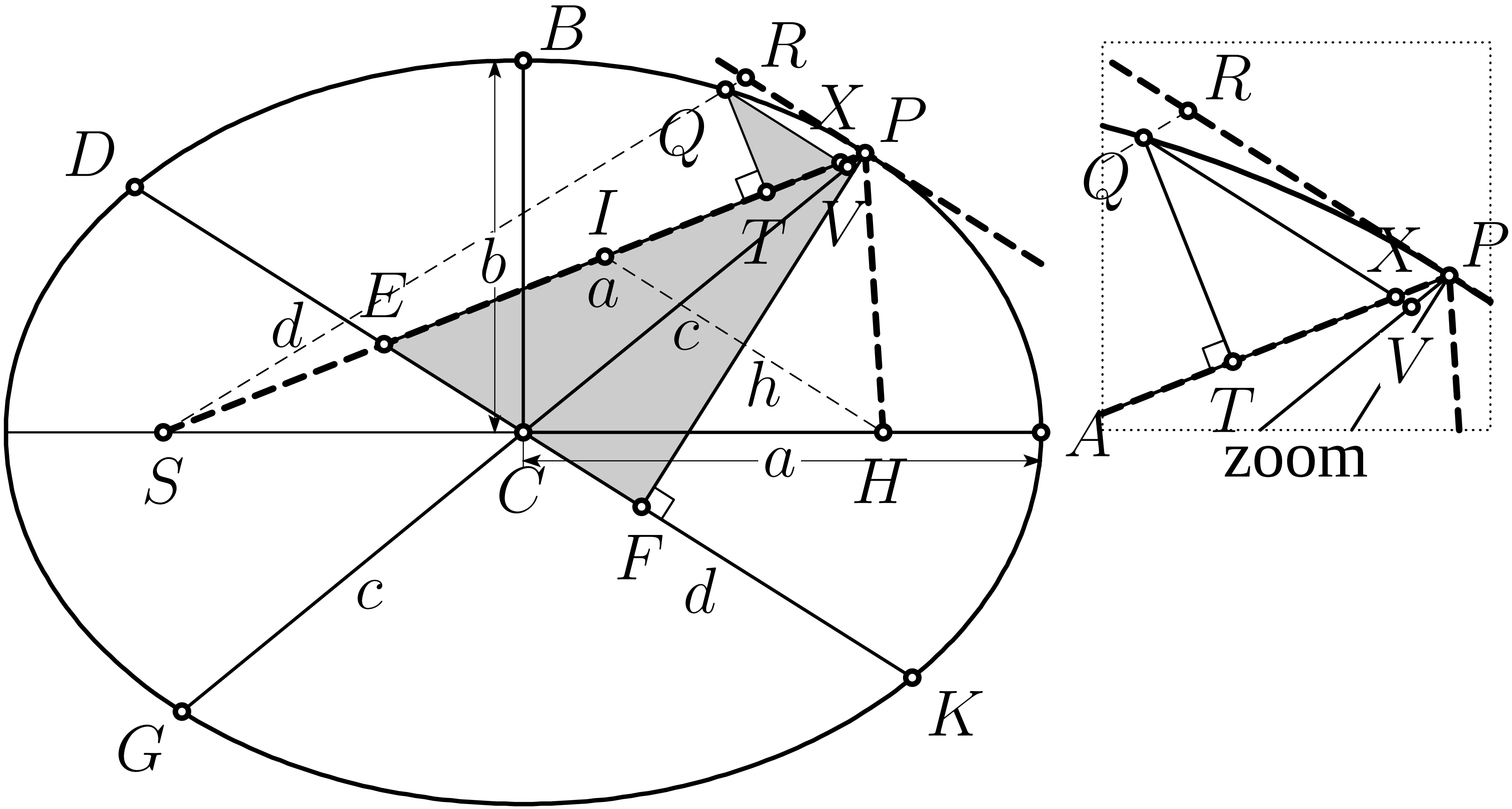
(Picture from Newton's Principia 1687)

Conj. diam. $\|$ tang.
(Apoll. II.6+Eucl. II.14)
$\Rightarrow PV/QV^2$ known

Tangents $\alpha = \alpha$
(Apoll. III.48)

$SP + HP = 2a$
(Apoll. III.52)

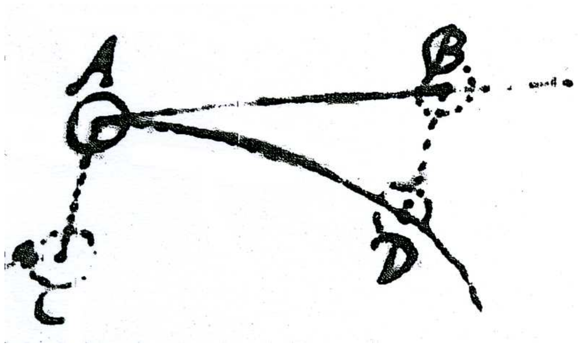
$\Rightarrow EP = a$ (Newton).



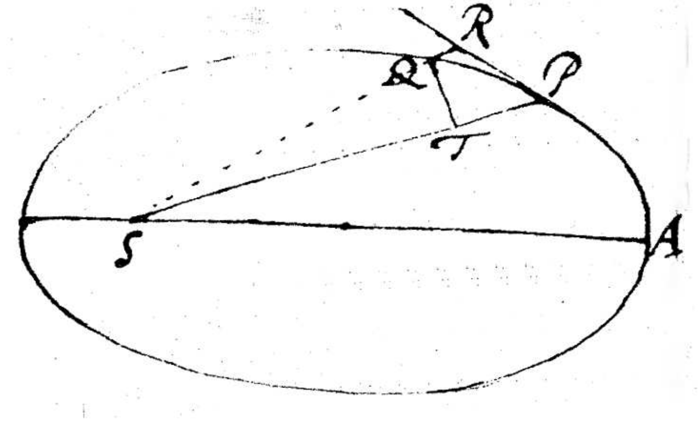
use similar triangles $PVX \sim PCE$ and $QTX \sim PFE$ and Apoll. VII.31 $(hd = ab)$

$$\Rightarrow \quad RQ \approx \frac{a}{2b^2} \cdot QT^2 \ .$$

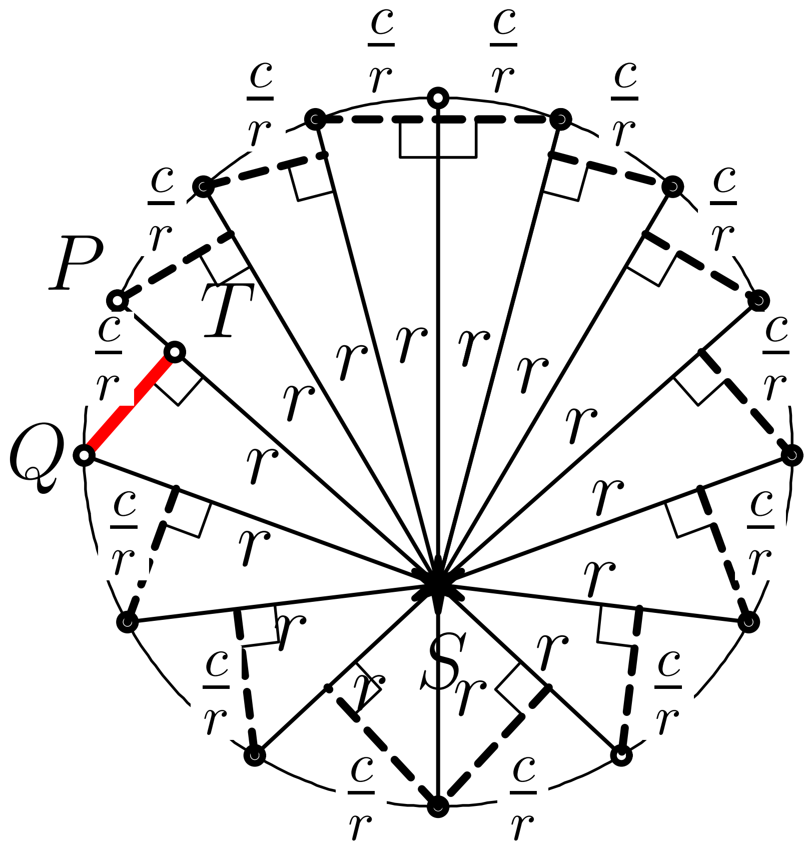
The Law of Gravitation.



force prop. BD
$= RQ$



RQ prop. QT^2
(Newton's Lemma)



QT prop. $\frac{1}{r}$
(Kepler 2)

hence: force is proportional to $\frac{1}{r^2}$

(Prop. XI of the Principia).

Another century later we arrive at Euler (E122) 1747:

$$\text{I. } \frac{2ddx}{dt^2} = \frac{X}{M}; \quad \text{II. } \frac{2ddy}{dt^2} = \frac{Y}{M}; \quad \text{III. } \frac{2ddz}{dt^2} = \frac{Z}{M}$$

for which the above algorithms are inverse numerical methods !!

"While physicists call these "Newton's equations", they occur nowhere in the work of Newton or of anyone else prior to 1747."

"...such is the universal ignorance of the true history of mechanics."

(C. Truesdell, *Essays in the History of Mechanics*, 1968)

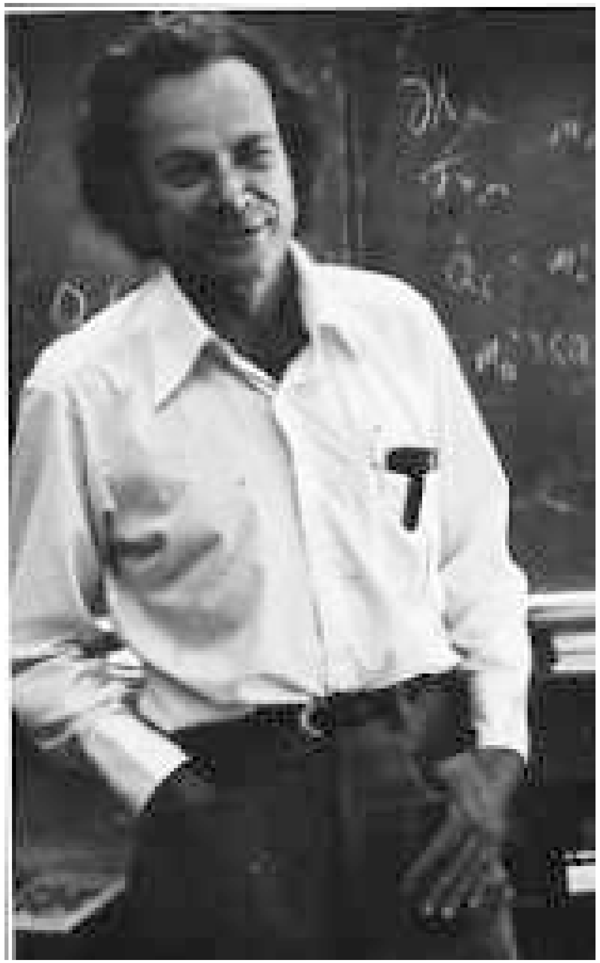
Feynman's solution of the inverse problem:

Inverse square law $\Rightarrow$ Kepler 1 ??

Pour voir présentement que cette courbe ABC ... est toûjours une Section Conique, ainsi que Mr. Newton l'a supposé, *pag. 55. Coroll.I.* sans le démontrer; il y faut bien plus d'adresse:

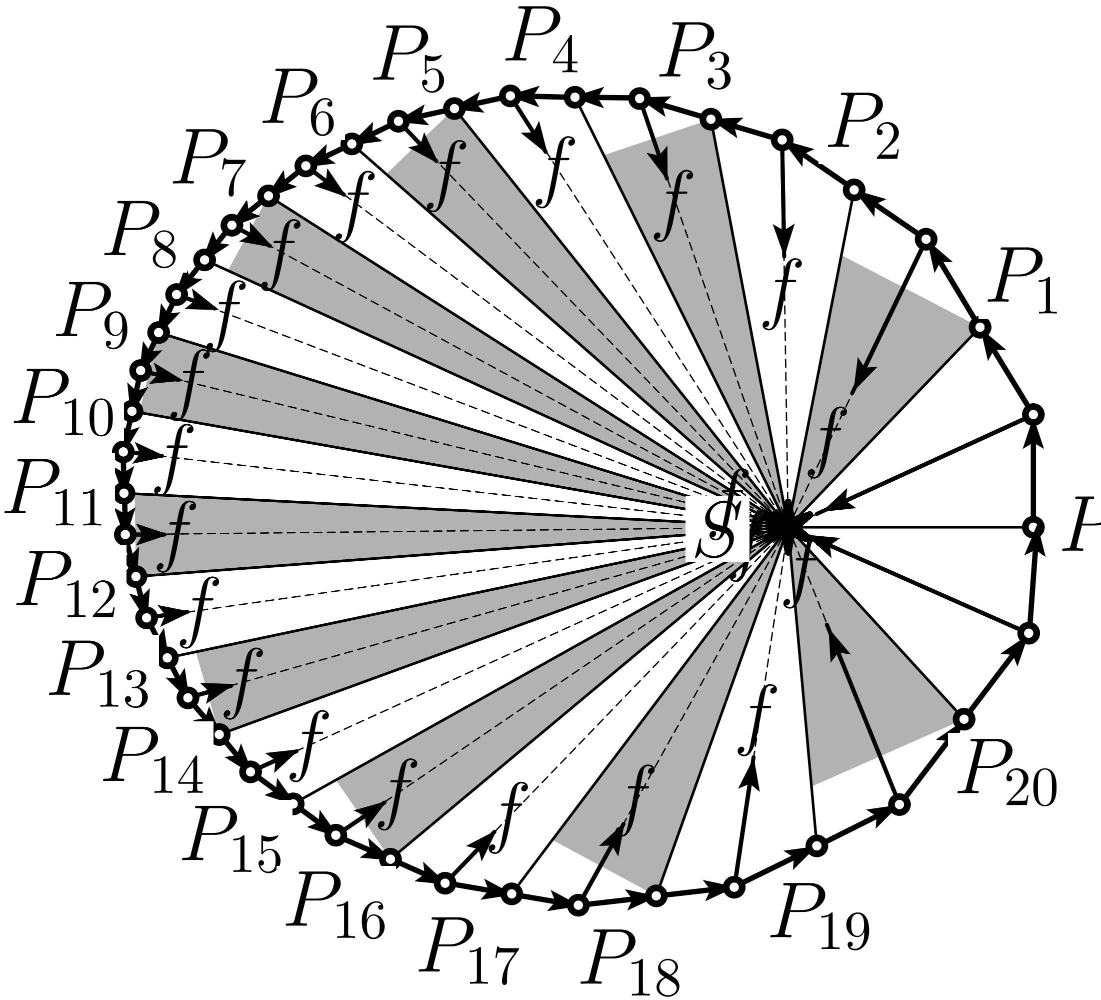
[To see that the orbit is always a conic section, which Newton claimed but did not prove, requires a good deal of more ability.]

(Joh. Bernoulli 1710)



Feynman's idea (1964, in a "lost lecture", rediscovered 1996):

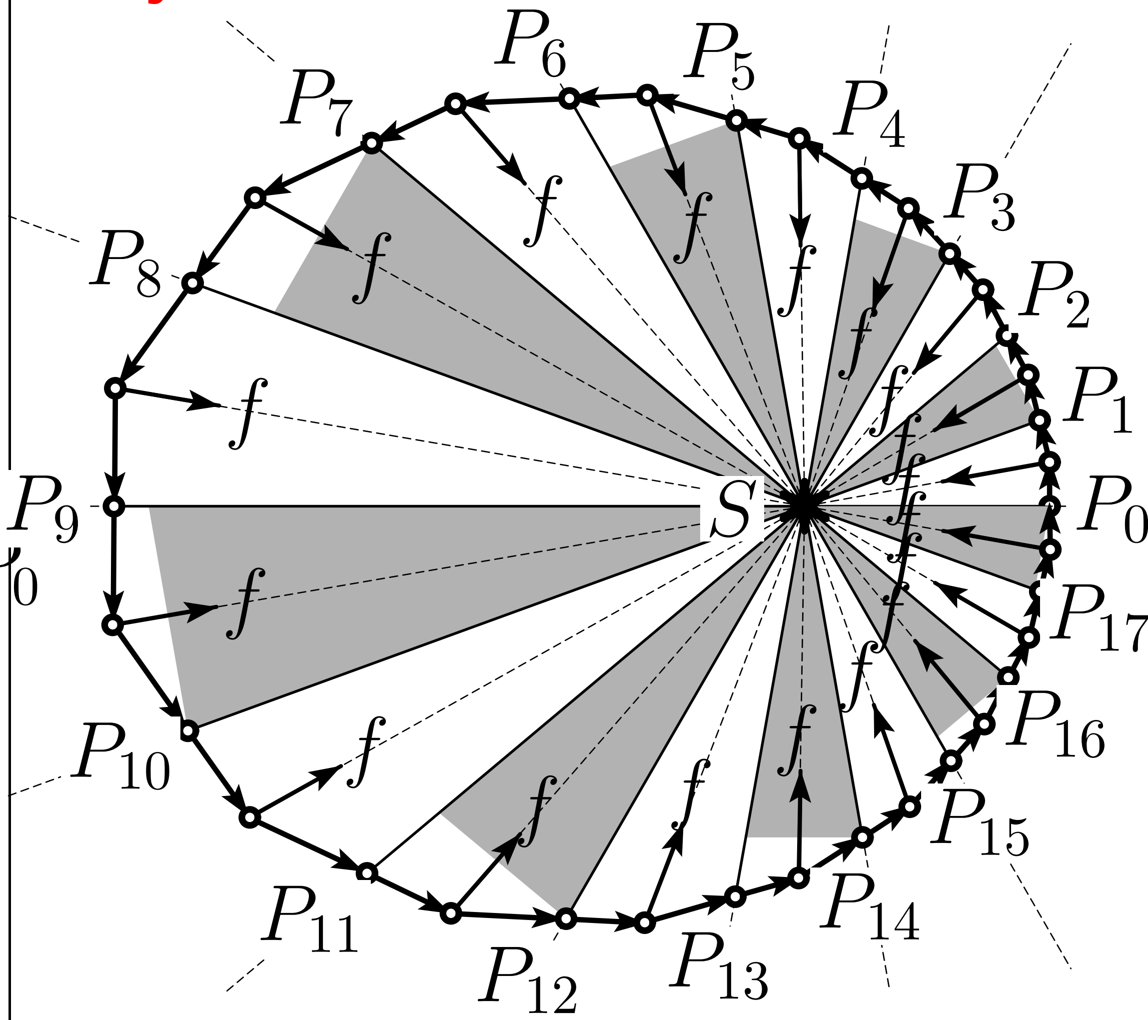
Kepler – Newton:



Δt constant

Force impulse prop. $\frac{1}{r^2}$.

Feynman:

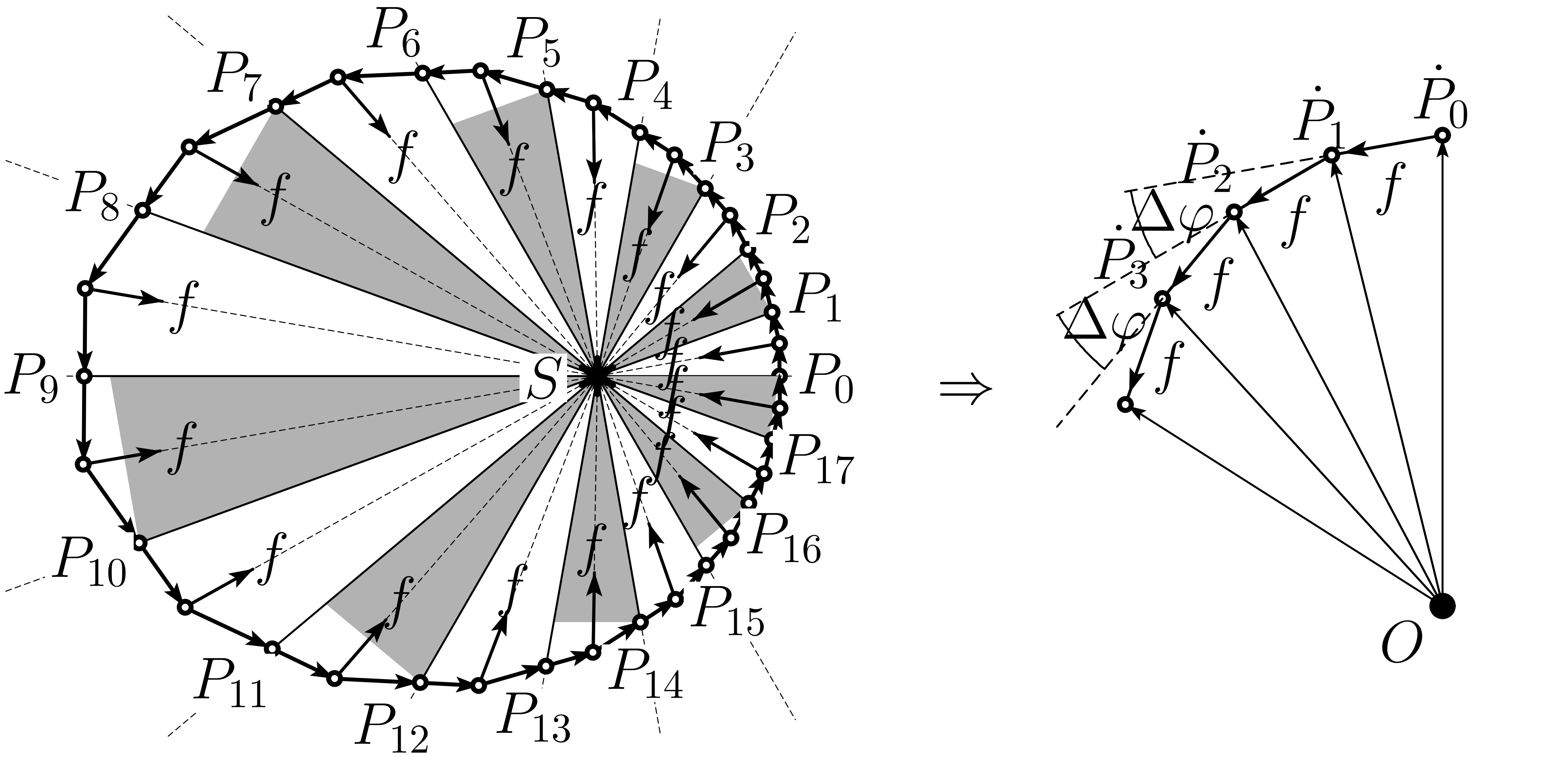


$\Delta\varphi$ constant

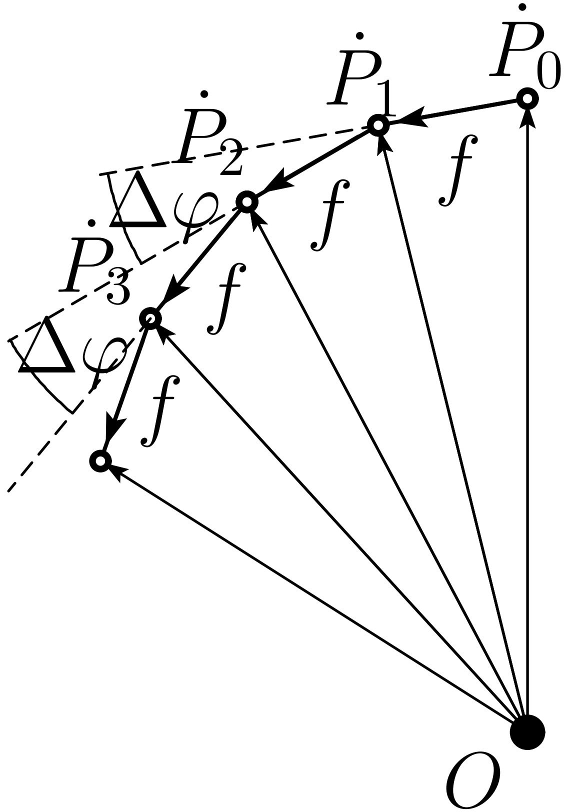
$\Rightarrow \Delta t$ prop. to r^2 (Eucl. VI.19);

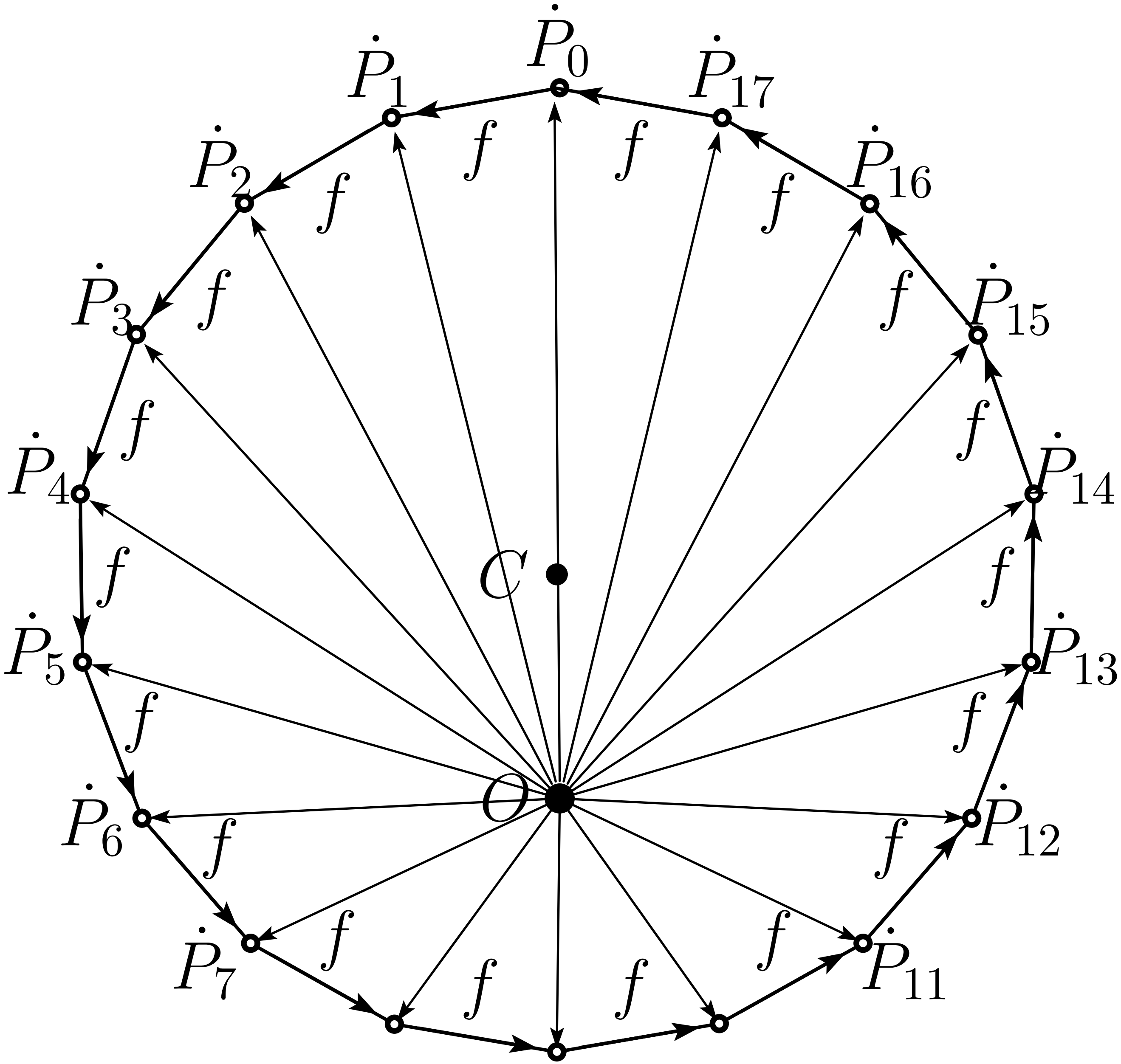
$\Rightarrow$ force impulses prop. 1 .

Look at hodograph (diagram of velocities)

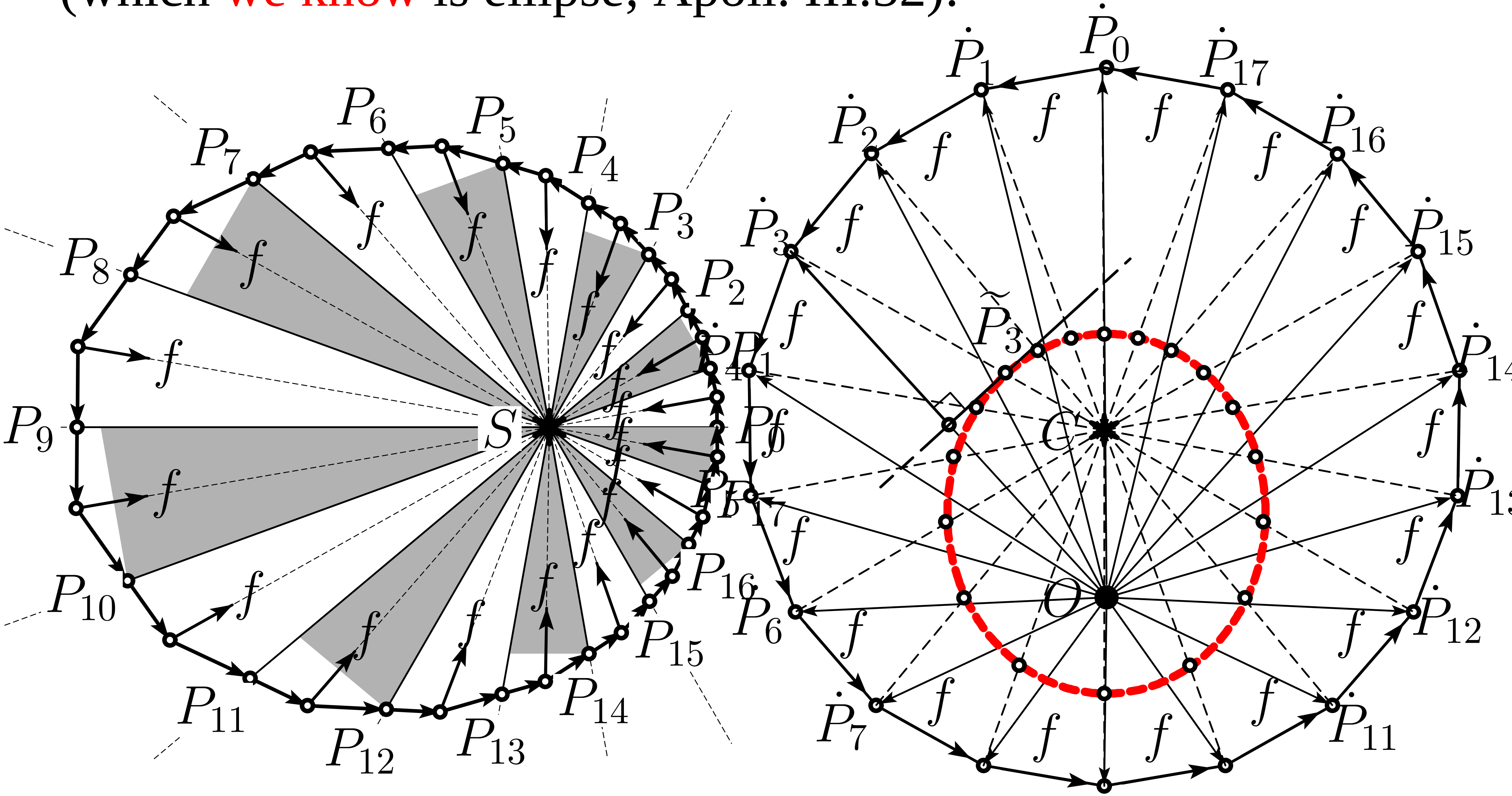


Velocity $\dot{P}$ of planet orbiting under inverse square force
describes a circle.





Idea: *Consider curve of same distance from circle and origin O*
(which we know is ellipse; Apoll. III.52):



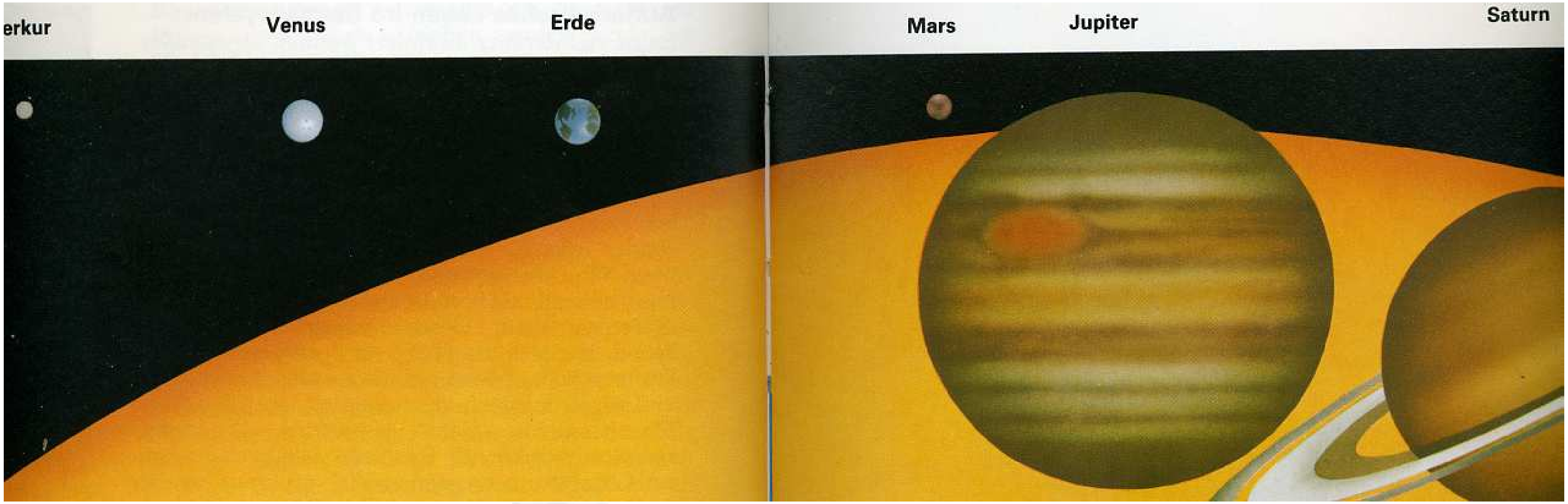
tangent in P_i parallel to $O\dot{P}_i$; tangent in $\widetilde{P}_i$ orthogonal to $O\dot{P}_i$;
$\Rightarrow$ **orbit is ellipse !!** □

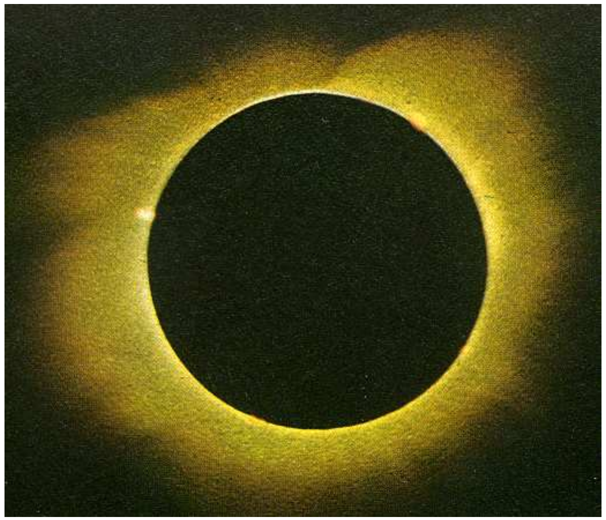
Épilogue ...

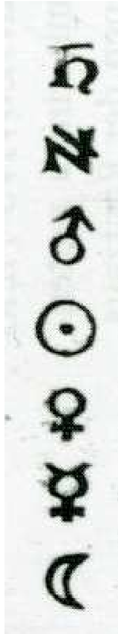
It is not easy to use the *geometric* method to discover things, it is very difficult, but the elegance of the demonstrations after the discoveries are made, is really very great. The power of the *analytic* method is that it is much easier to discover and to prove things, but not in any degree of elegance. There is a lot of dirty paper with x-es and y-s and crossed out cancellations and so on ... (laughers).

(R. Feynman, lecture of march 13, 1964, 35th minute.)

PART II: Find relations between **all** planets.







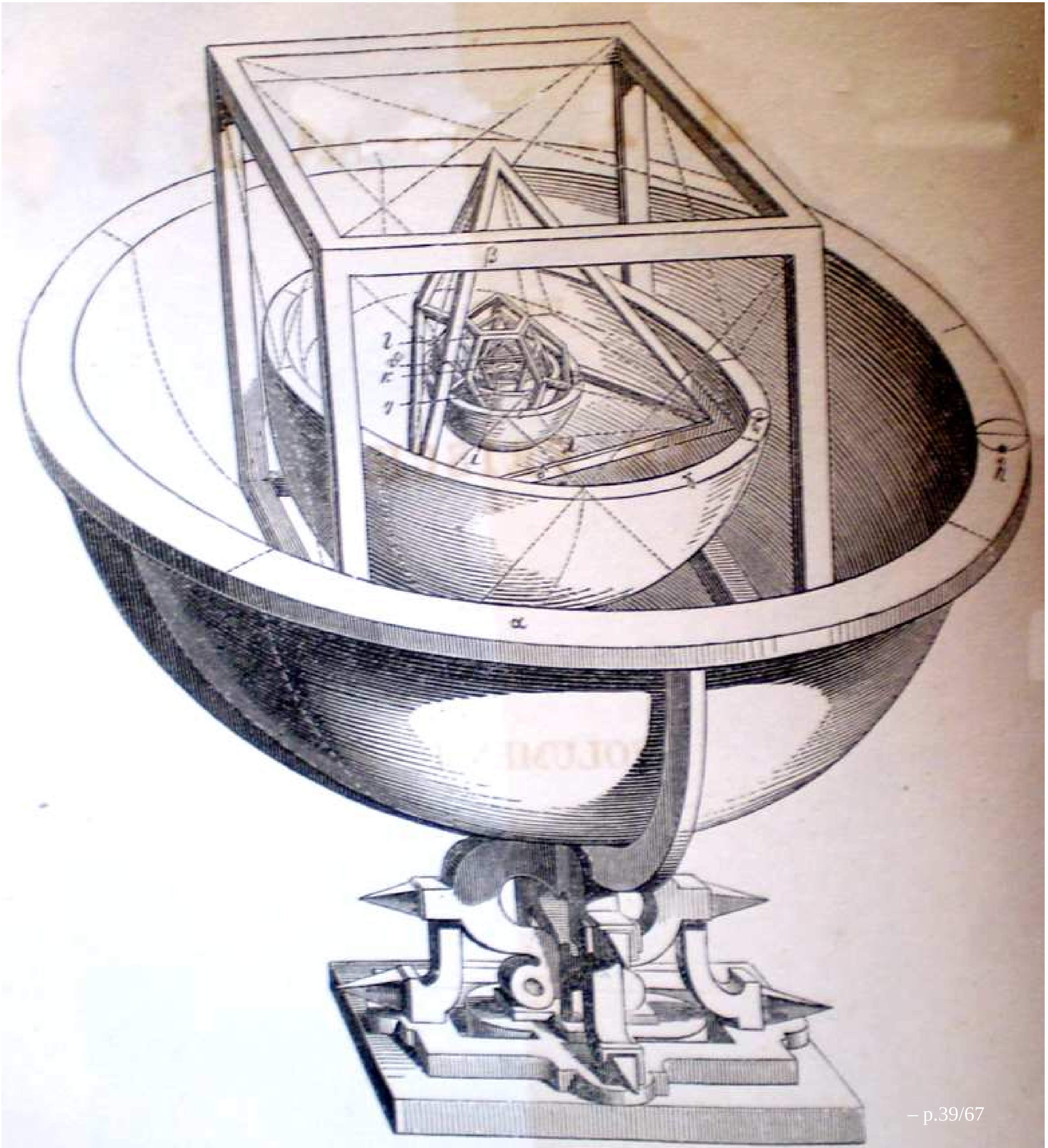
Babylon. and Egypt. civilisation:

The Seven Heavenly Gods

Sunday, lunedi, martedi, mercoledi, giovedi, venerdi, Saturday.

Johannes Kepler

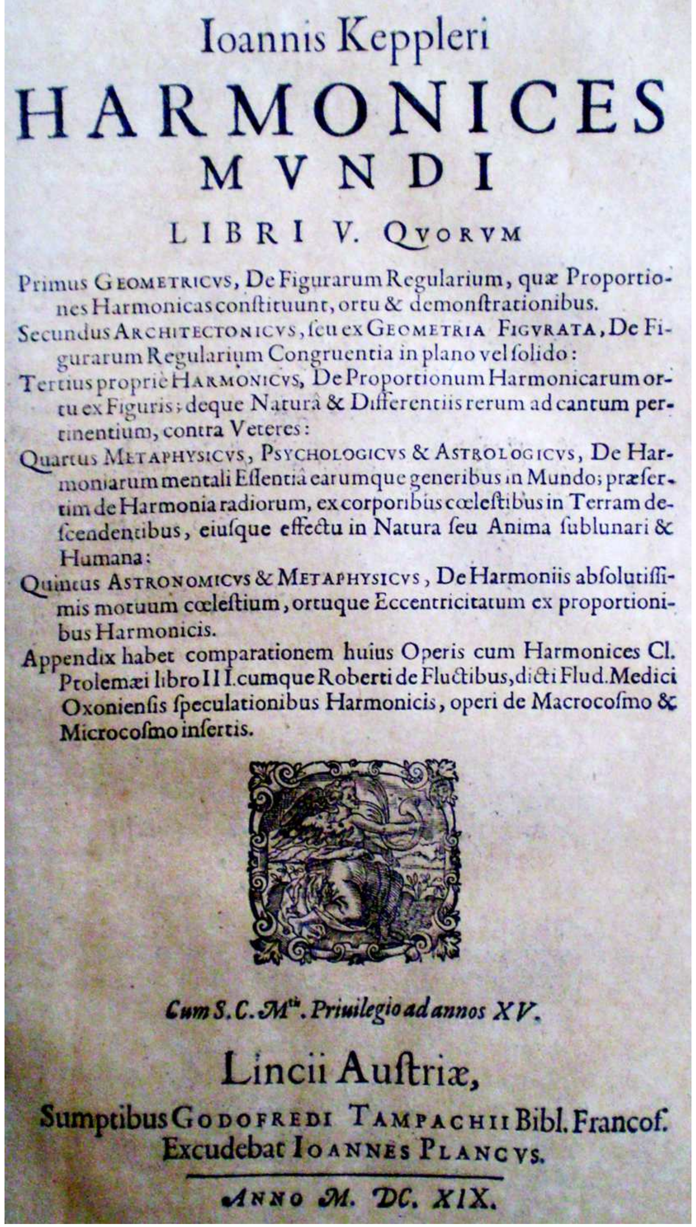
Mysterium Cosmographicum (1596)



69 Thomas Lansius: Anekdote über Kepler

Joan. Harprechtvs, Antecessor in Academia Tubingensi, singularis exempli: suprema laudatione celebratus a Thoma Lansio. Tubingae, Typis Brunnianis 1640 – UB: L XVI 129. 4° angeb. – In der Trauerrede auf Harprecht (S. 56 f.) erzählt Lansius: „Als ich Kepler, der mir in wahrer Freundschaft verbunden war . . ., einmal fragte, welches von seinen Werken er am höchsten schätze, gab er den Primat dem ‚Mysterium cosmographicum' und bezeugte, daß in dieser Schrift das erhabene Geheimnis der fünf regulären Körper nach vielhundertjähriger Verborgenheit . . . eröffnet werde. Als diese Entdeckung noch neu gewesen sei, habe er sie so hoch geschätzt, daß er, wenn ihm gleichzeitig die Kurfürstenwürde Sachsens unter der Bedingung zu Geschenk angeboten worden wäre, entweder das Geschenk oder die Entdeckung zu verschmähen, lieber diese große und an Bergwerken reiche Provinz verlieren als die beneidenswerte und ewigen Ruhm mit sich bringende Entdeckung hätte entbehren wollen."

Kepler's Third Law (1619):



Ioannis Keppleri

HARMONICES MVNDI

LIBRI V. QVORVM

Primus GEOMETRICVS, De Figurarum Regularium, quæ Proportiones Harmonicas constituunt, ortu & demonstrationibus.

Secundus ARCHITECTONICVS, seu ex GEOMETRIA FIGVRATA, De Figurarum Regularium Congruentia in plano vel solido:

Tertius propriè HARMONICVS, De Proportionum Harmonicarum ortu ex Figuris; deque Naturâ & Differentiis rerum ad cantum pertinentium, contra Veteres:

Quartus METAPHYSICVS, PSYCHOLOGICVS & ASTROLOGICVS, De Harmoniarum mentali Essentiâ earumque generibus in Mundo; præsertim de Harmonia radiorum, ex corporibus cœlestibus in Terram descendentibus, eiusque effectu in Natura seu Anima sublunari & Humana:

Quintus ASTRONOMICVS & METAPHYSICVS, De Harmoniis absolutissimis motuum cœlestium, ortuque Eccentricitatum ex proportionibus Harmonicis.

Appendix habet comparationem huius Operis cum Harmonices Cl. Ptolemæi libro III. cumque Roberti de Fluctibus, dicti Flud. Medici Oxoniensis speculationibus Harmonicis, operi de Macrocosmo & Microcosmo insertis.

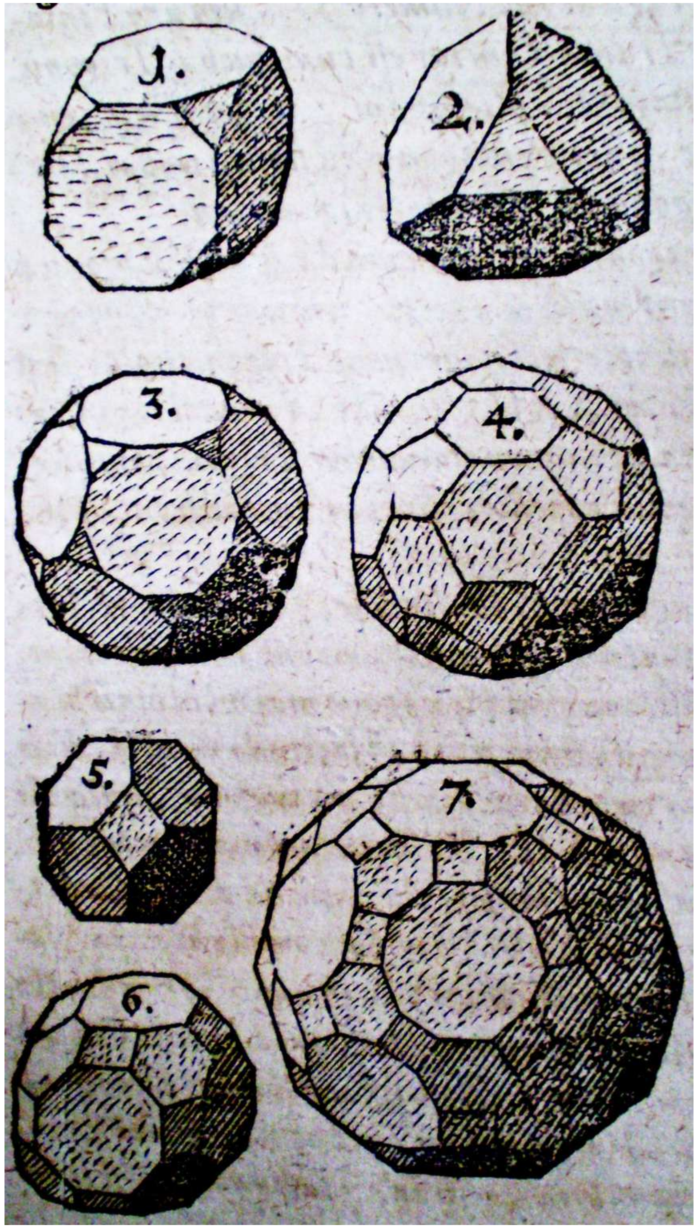
Cum S. C. M^tis. Privilegio ad annos XV.

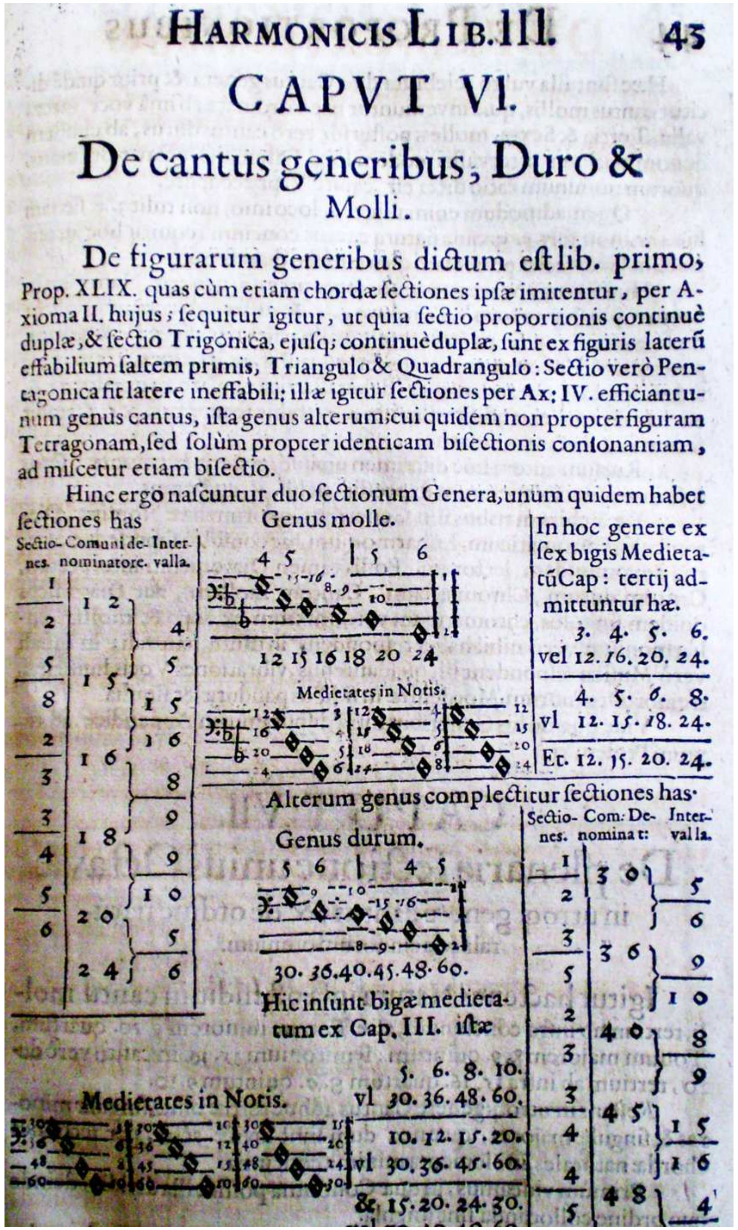
Lincii Austriæ,

Sumptibus GODOFREDI TAMPACHII Bibl. Francof.

Excudebat IOANNES PLANCVS.

ANNO M. DC. XIX.





HARMONICIS LIB. III. 43

CAPVT VI.

De cantus generibus, Duro & Molli.

De figurarum generibus dictum est lib. primo; Prop. XLIX. quas cùm etiam chordæ sectiones ipsæ imitentur, per Axioma II. hujus; sequitur igitur, ut quia sectio proportionis continuè duplæ, & sectio Trigonica, ejusq; continuè duplæ, sint ex figuris laterû effabilium saltem primis, Triangulo & Quadrangulo: Sectio verò Pentagonica fit latere ineffabili; illæ igitur sectiones per Ax: IV. efficiant unum genus cantus, ista genus alterum: cui quidem non propter figuram Tetragonam, sed solùm propter identicam bisectionis consonantiam, admiscetur etiam bisectio.

Hinc ergò nascuntur duo sectionum Genera, unum quidem habet sectiones has

Genus molle.

In hoc genere ex sex bigis Medietatû Cap: tertij admittuntur hæ.

Medietates in Notis.

Alterum genus complectitur sectiones has:

Genus durum.

Hic insunt bigæ medietatum ex Cap. III. istæ

Medietates in Notis.

Search harmonies... from Geometry... and Music...

... Finally the result comes from vulgar numer. calculations ...

	Diurni Prim. Sec.	Intervalla mediocria	Itinera diurna.
Saturni Aphelij	1. 53.	9510.	1065
Perihelij	2. 7.		1208
Iovis Aphelij	4. 44	5200.	1477
Perihelij	5. 15.		1638
Martis Aphelij	28. 44.	1524.	2627
Perihelij	34. 34.		3161
Telluris Apheliæ	58. 6.	1000.	3486
Periheliæ	60. 13.		3613
Veneris Apheliæ	95. 29.	724	4149
Periheliæ	96. 50.		4207
Mercurij Aphelij	201. 0.	388.	4680
Perihelij	307. 3.		7148

inter p...cipia, primò crederem. Sed res est certissima exactissimaque, quòd *proportio quæ est inter binorum quorumcunque Planetarum tempora periodica, sit præcisè sesquialtera proportionis* mediarum distantiarum, id

"It is extremely certain and extremely exact that the ratio of the time period for two planets is one and a half of the ratio of the mean distances" (Lib. V, Caput 3, § 8).

The Rule of Titius-Bode.

> Das Daseyn dieses Planeten scheint insbesondere aus einem merkwürdigen Verhältniss zu folgen . . .
>
> (J.E. Bode, *Anleitung zur Kenntniss des gestirnten Himmels*, 6. Aufl., Berlin 1792, quoted in *Hegels Werke* 5, Anmerkungen p. 810)

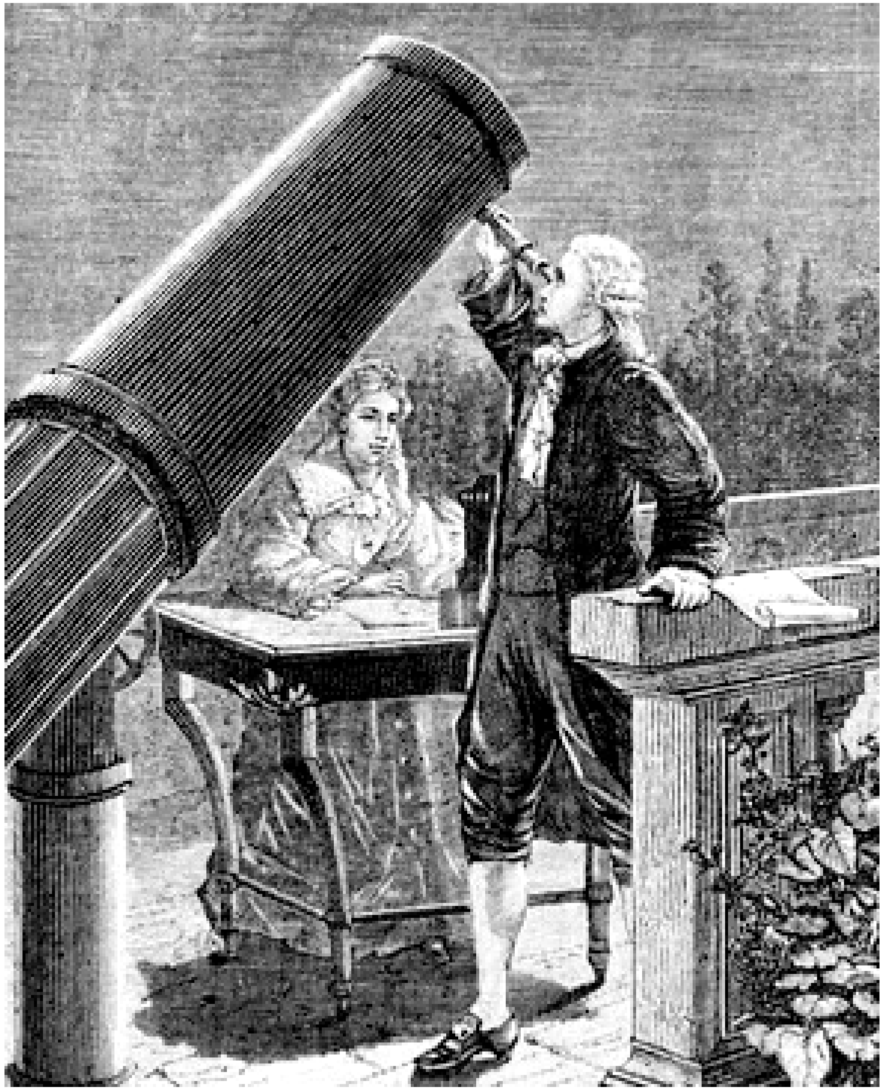
$$0.4, \quad 0.4 + 0.3 = 0.7, \quad 0.4 + 2 \cdot 0.3 = 1 \text{ (the earth)}, \quad \ldots$$

$$0.4 + 2^{n-2} \cdot 0.3, \ldots \; .$$

For $n = 1, 2, 3, 4, 6, 7$ approximate well the distances of the known planets.

$n = 5$ is missing !! Is there a gap??

“Sollte der Urheber der Welt diesen Raum leer gelassen haben?”



Discovery of Uranus.

Sir William Herschel,
a German organist
and amateur astronomer
living in England, discovered
the 13th of march 1781
a new planet
through a huge telescope
of his own construction.

Herschel wanted to name ‘his’ new planet *Georgium sidus* (George’s star), in devotion to the British King, but Bode’s proposition *Uranus* (in Greek mythology the father of Saturnus), was felt less patriotic.

Also Uranus fitted well, for $n = 8$, into Bode’s formula.

The Thesis of Hegel.

Dissertatio philosophica de orbitis planetarum, Ienae MDCCCI.

Plato's *Timæus*: the "Soul of the World" :

Moon	1
Sun	2
Venus	3
Mercury	4
Mars	8
Jupiter	9
Saturn	27

$\sqrt[3]{x^4}$ $\Rightarrow$ 1.000 2.520 4.327 6.350 16.000 18.721 81.000

"16 enim pro 8 quem legimus ponere liceat" ...ponamus $1 = \sqrt[3]{3}$

1.4 2.56 4.37 6.34 18.75 40.34 81

"inter quartum et quintum locum magnum esse spatium"

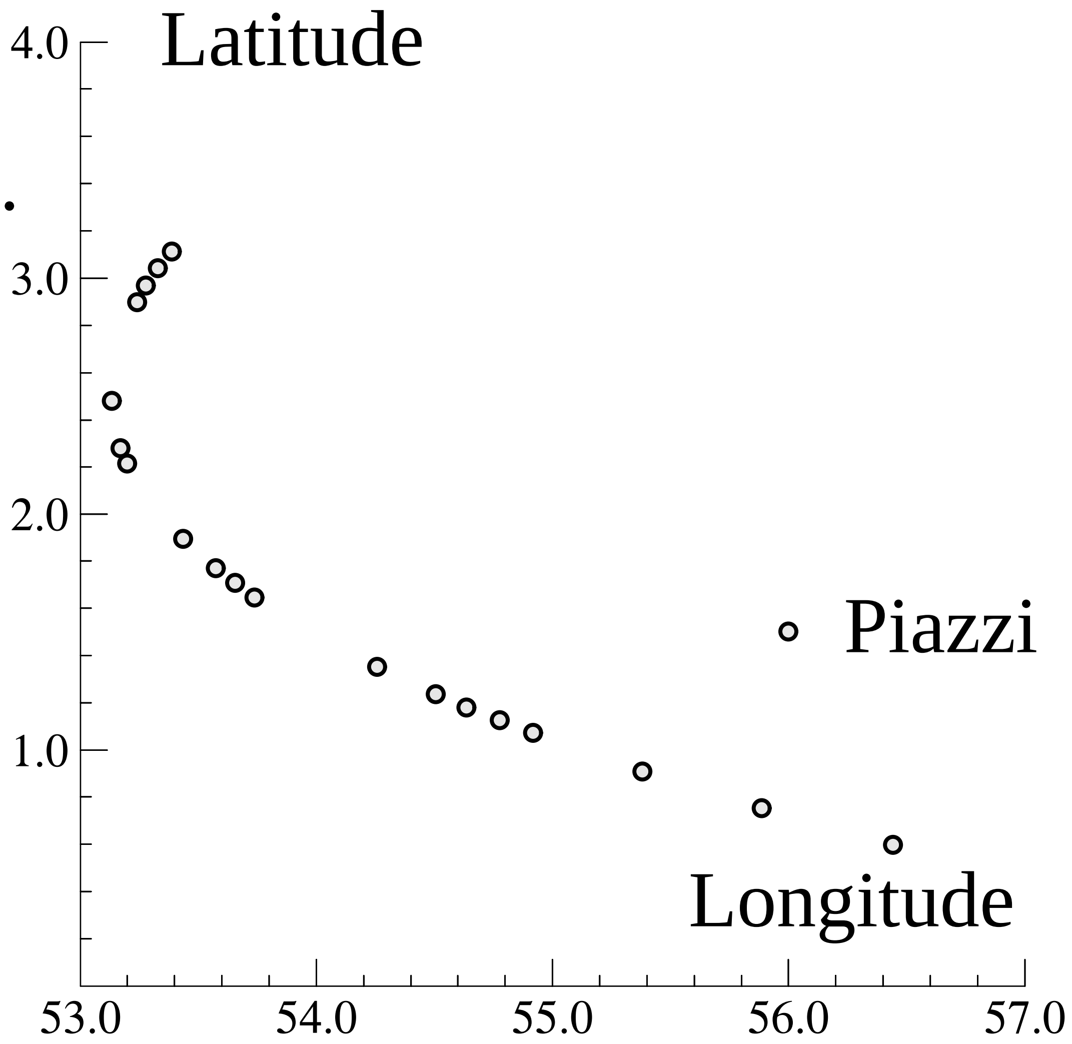
Philosophical "proof" that NO PLANET IS MISSING!!

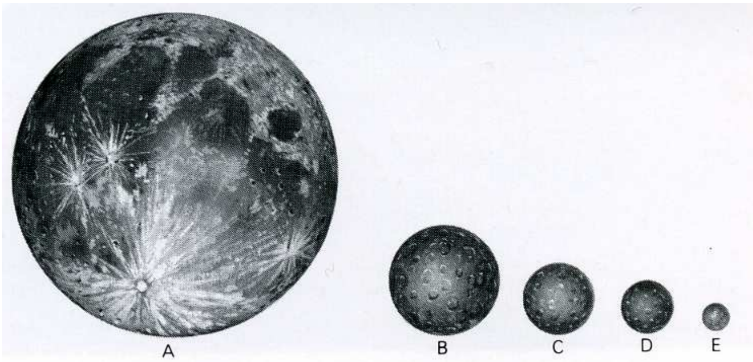
Sehen Sie sich doch nur bei den heutigen Philosophen um, bei Schelling, Hegel, Nees von Esenbeck und Consorten, stehen Ihnen nicht die Haare bei ihren Definitionen zu Berge?

(Brief von Gauss an Schumacher, 1. 11. 1844)

The Discovery of Piazzi.

On January 1, 1801, the Italian astronomer Giuseppe Piazzi discovered in the Taurus constell. a tiny little spot, followed its orbit until the 11th of February, when illness, bad weather, and the approaching Sun interrupted the observations. He named it *Ceres Ferdinandea* (Ferdinand is another King).





PROBLEM: Find this planet again!!

1801	Longitude	Latitude		Longitude	Latitude
Jan. 1	$53^0\ 23'\ 06.38''$	$3^0\ 06'\ 45.16''$	23	$53^0\ 44'\ 12.46''$	$1^0\ 38'\ 46.78''$
2	$53^0\ 19'\ 38.18''$	$3^0\ 02'\ 26.46''$	28	$54^0\ 15'\ 18.52''$	$1^0\ 21'\ 04.92''$
3	$53^0\ 16'\ 37.70''$	$2^0\ 58'\ 08.04''$	30	$54^0\ 30'\ 10.52''$	$1^0\ 14'\ 14.24''$
4	$53^0\ 14'\ 21.44''$	$2^0\ 53'\ 51.98''$	31	$54^0\ 38'\ 05.58''$	$1^0\ 10'\ 51.02''$
10	$53^0\ 07'\ 57.64''$	$2^0\ 28'\ 53.64''$	Feb. 1	$54^0\ 46'\ 27.14''$	$1^0\ 07'\ 34.18''$
13	$53^0\ 10'\ 05.60''$	$2^0\ 16'\ 46.08''$	2	$54^0\ 55'\ 01.52''$	$1^0\ 04'\ 18.10''$
14	$53^0\ 11'\ 54.20''$	$2^0\ 12'\ 54.02''$	5	$55^0\ 22'\ 45.20''$	$0^0\ 54'\ 34.54''$
19	$53^0\ 26'\ 01.98''$	$1^0\ 53'\ 37.82''$	8	$55^0\ 53'\ 04.52''$	$0^0\ 45'\ 08.28''$
21	$53^0\ 34'\ 22.68''$	$1^0\ 46'\ 13.06''$	11	$56^0\ 26'\ 28.20''$	$0^0\ 35'\ 55.02''$
22	$53^0\ 39'\ 11.58''$	$1^0\ 42'\ 28.80''$			

The observations of Piazzi

Many astronomers took part at the great challenge of the re-discovery (Burckhardt, Olbers, Piazzi).

But a certain, 24 years old, “Dr. Gauss in Braunschweig” computed a different solution “nach einem eigenthümlichen Verfahren” and published it the 29th of Sept. 1801.

Still better solution in December 1801:

Sonnenferne	$326^0\ 53'\ 50''$
Ω	$81^0\ 1'\ 44''$
Neigung der Bahn	$10^0\ 36'\ 21''$
Logarithmus der halben grossen Axe	0.4414902
Excentricität	0.0819603
Epoche: 31 Dec. 1800 mittl. helioc. Länge	$77^0\ 54'\ 29''$

The 7th of December 1801, Freiherr von Zach re-discovered Ceres precisely at the position predicted by Gauss.

How did Gauss compute his solution?

Gewiss, jeder der die Rechnungen kennt, die die Bestimmung der Elemente eines Planeten und dann jeder daraus herzuleitende Ort erfordert, muss es bewundern, wie ein einzelner Mann in so kurzen Zeiträumen so vielfache mühsame Rechnungen zu vollenden vermögend war.

(von Zach, März 1805, see Gauss *Werke* 6, p. 262)

Published posthumely; Werke vol. 11, pp. 221-252, and vol. 6, p. 199–402.

BESTIMMUNG DER BAHNEN DER HIMMELSKÖRPER.

[Aus Handbuch 16, Bb, Den astronomischen Wissenschaften gewidmet, November 1801, S. 1—8.]

[1.]

Bedeutung der hier vorkommenden Zeichen.

α ... Geocentrische Länge des Weltkörpers	t ... Zeit der Beobachtung
	v ... Länge in der Bahn
β ... Geocentrische Breite	M ... Wahre Anomalie [*)]
θ ... Tangente der Breite	E ... Eccentrische Anomalie [*)]
Δ ... Kurtirter Abstand von der Erde	N ... Mittlere Anomalie [*)]
λ ... Heliocentrische Länge	u ... Mittlere Länge
β ... Heliocentrische Breite	R ... Abstand der Sonne von der Erde
ϑ ... Tangente dieser Breite	V ... Wahre Länge der Sonne
ρ ... Kurtirter Abstand von der Sonne	U ... Mittlere Länge der Sonne.
r ... Wahrer Abstand von der Sonne	

Dieselben Zeichen mit Linien haben ähnliche Bedeutungen für andere
achtungen.

. Länge des aufsteigenden Knoten
. Neigung der Bahn
. Tangente derselben
. Ort der Sonnenferne
. Eccentricität
. Bogen dessen Sinus $= e$
. Halbe grosse Axe
. Halbe kleine Axe
. Halber Parameter

Anm. Die Längen sind sämmtlich siderisch, oder von einem festen, übrigens willkürlichen Punkte des Himmels an gezählt.

[2.]

Formeln zur ersten Annäherung aus dreien Beobachtungen.

$$\frac{R'}{\Delta'}\left(1-\frac{R'^3}{r'^3}\right)=\frac{\theta\sin(\alpha''-\alpha')-\theta'\sin(\alpha''-\alpha)+\theta''\sin(\alpha'-\alpha)}{\frac{1}{2}(U'-U)(U''-U')\{\theta''\sin(V'-\alpha)-\theta\sin(V'-\alpha'')\}}\text{[*)]},$$

a.] genau $$\left[\frac{1}{\Delta' R' R'}\left(1-\frac{R'^3}{r'^3}\right)\right]=\frac{d\alpha'\,dd\theta'-d\theta'\,dd\alpha'+\theta'\,d\alpha'^3}{dU^2\{\theta'\cos(V'-\alpha')\,d\alpha'+\sin(V'-\alpha')\,d\theta'\}}.$$

Hat man nach dieser Formel $\frac{R'}{\Delta'}\left(1-\frac{R'^3}{r'^3}\right)$ bestimmt, so findet man dar-
leicht durch Verbindung mit der Gleichung

$$\frac{r'}{\Delta'}=\sqrt{\left(1+\theta'\theta'+\frac{R'R'}{\Delta'\Delta'}-2\frac{R'}{\Delta'}\cos(V'-\alpha')\right)}$$

ittelst weniger Versuche einen nahen Werth von Δ'.

$$\frac{\Delta''}{\Delta}=\frac{\theta'\sin(V'-\alpha)-\theta\sin(V'-\alpha')}{\theta''\sin(V'-\alpha')-\theta'\sin(V'-\alpha'')}\cdot\frac{t''-t'}{t'-t}=\frac{\frac{\sin(V'-\alpha)}{\sin(V'-\alpha')}-\frac{\theta}{\theta'}}{\frac{\theta''}{\theta'}-\frac{\sin(V'-\alpha'')}{\sin(V'-\alpha')}}\cdot\frac{t''-t'}{t'-t}\text{[**)]}.$$

g genau ist

b.] $$\frac{2d\Delta'}{\Delta'}=-\frac{\theta'\,dd\alpha'\cos(V'-\alpha')+(\theta'\,d\alpha'^2+dd\theta')\sin(V'-\alpha')}{\theta'\,d\alpha'\cos(V'-\alpha')+d\theta'.\sin(V'-\alpha')}.$$

In Ansehung der ersten Formel ist noch zu bemerken, dass $U'-U$,

[*) Vgl. Werke VI, S. 159.]
[**) Vgl. Werke VI, S. 157.]

$U''-U'$ in Theilen des Radius ausgedrückt werden müssen; auf die
ist log (mot. med. ⊙ diurn.) = 8,235 5820.792[*)].

Nachdem man nun Δ' und $\frac{\Delta''}{\Delta}$ bestimmt hat, kann man, hinreiche
nau zur ersten Näherung

$$\log\Delta=\log\Delta'-\frac{t'-t}{t''-t}\log\frac{\Delta''}{\Delta},\qquad \log\Delta''=\log\Delta'+\frac{t''-t'}{t''-t}\log\frac{\Delta''}{\Delta}$$

setzen.

Ist die Neigung der Bahn sehr gering, so sind obige Gleichung
brauchbar; die beobachtete Länge, ihre Veränderung und deren Zu
Abnahme gibt sodann bloss folgende Gleichung:

[c.] $$0=2\frac{d\Delta}{dU}\frac{d\alpha}{dU}+\Delta\frac{d^2\alpha}{dU^2}+R\left(\frac{A^3}{R^3}-\frac{A^3}{r^3}\right)\sin(V-\alpha).$$

[Im übrigen ist:]

[d.] $$0=\theta.\Delta\cdot\frac{d\alpha^2}{dU^2}+2\frac{d\Delta}{dU}\frac{d\theta}{dU}+\Delta\frac{d^2\theta}{dU^2}-R\theta\left(\frac{A^3}{R^3}-\frac{A^3}{r^3}\right)\cos(V-\alpha).$$

[S. 3]

[3.]

Vorschriften zur Berechnung der Elemente, aus zweien geoc
schen Örtern, der Zwischenzeit, und den zugehörigen Abstände

$$\rho=\sqrt{\{RR+\Delta\Delta-2R\Delta\cos(V-\alpha)\}}.$$

Ganz allgemein

$$\left\{\begin{matrix}\Delta\sin\alpha-R\sin V=\rho\sin\lambda\\ \Delta\cos\alpha-R\cos V=\rho\cos\lambda\end{matrix}\right\};$$

folglich

$$\text{I.}\left\{\begin{matrix}\Delta\sin(V-\alpha)=\rho\sin(V-\lambda)\\ \Delta\cos(V-\alpha)-R=\rho\cos(V-\lambda)\end{matrix}\right\},\qquad \text{II.}\left\{\begin{matrix}R\sin(V-\alpha)=\rho\sin(\alpha-\lambda)\\ R\cos(V-\alpha)-\Delta=-\rho\cos(\end{matrix}\right.$$

$$\text{III.}\left\{\begin{matrix}(\Delta+R)\sin\frac{1}{2}(V-\alpha)=\rho\sin\{\frac{1}{2}(V+\alpha)-\lambda\}\\ (\Delta-R)\cos\frac{1}{2}(V-\alpha)=\rho\cos\{\frac{1}{2}(V+\alpha)-\lambda\}\end{matrix}\right\}$$

[*)] Wenn man auf die Erdmasse mit Rücksicht nimmt, muss man log mot
Planetae in distantia media setzen: $8{,}235\,5814.21\cdot[A^{\frac{3}{2}}:a^{\frac{3}{2}}$, wo wie auch im Fo
A die halbe große Achse der Erdbahn bedeutet].

[**) Vgl. Theoria motus, art. 124, Werke VII, 1906, S. 165.]

$-)$, $\quad \frac{e}{p}\cos\left(\tfrac12(v''+v)-\omega\right) = \dfrac{\frac1p-\frac12\left(\frac1{r''}-\frac1r\right)}{\cos\frac12(v''-v)}$,

$\frac{p}{\cos\varphi}$, $\quad a = \frac{p}{\cos\varphi^2}$,

med. $= \dfrac{\text{mot. diurn.} \odot \text{med.}}{a^{\frac32}}$,

$-\omega$, $\quad M'' = v'' - \omega$,

$\sqrt{\frac{r}{p}}\}$ $\quad \operatorname{tg}\frac12 E = \operatorname{tg}\frac12 M \operatorname{tg}(\frac12\varphi+45^0)$,

zur Probe

$\sqrt{\frac{r''}{p}}\}$ $\quad \operatorname{tg}\frac12 E'' = \operatorname{tg}\frac12 M'' \operatorname{tg}(\frac12\varphi+45^0)$,

$\Xi'' = \sin\varphi\sin M''\cdot\frac{r''}{b}$, $\quad t''-t = \dfrac{N''-N}{\text{mot. diurn. med.}}$]

693 $\quad [\frac12(rr+r''r'')\ldots]$ 0,856 1994

071 $\quad \left[\frac{U''-U}{v''-v}\ldots\right]$ 0,638 3472

764 $\quad [(\sqrt{p})\ldots]$ 0,217 8522

292 $\quad [(p)\ldots]$ 0,435 7044

472 $\quad \left[\sqrt[3]{\left(\frac{2\Re\Re}{rr+r''r''}\right)^4}\ldots\right]$ $-$ 1223 [*)]

Parameter $[p\ldots]$ 0,435 5821

$\sin\frac14(v''-v)^2\ldots]$ 7,514 4812 $\quad \left[\frac12\left(\frac1r+\frac1{r''}\right)-\frac1p\ldots\right]$ 7,805 6761

$\cos\frac12(v''-v)\ldots]$ 9,998 5777 $\quad \left[\frac{2\sin\frac14(v''-v)^2}{\cos\frac12(v''-v)}\ldots\right]$ 7,515 9035

[Diff.] 7,515 9035 $\quad$ [Summe ✱] 5,321 58

$[\Re\ldots]$ 0,428 05386

$[\Re\Re\ldots]$ 0,856 1077

8 $\quad [\frac12(rr+r''r'')\ldots$ 0,856 1]994

9 $\quad \left[\frac12\,\frac{rr+r''r''}{\Re\Re}\ldots\right.$ 0,000 0]917

7 $\quad \left[\sqrt[3]{\left(\frac12\,\frac{rr+r''r''}{\Re\Re}\right)}\ldots\right.$ 0,000 0]306

$\quad \left[\sqrt[3]{\left(\frac12\,\frac{rr+r''r''}{\Re\Re}\right)^4}\ldots\right.$ 0,000 1223]

...ehenden letzten Annäherung zur Bestimmung von p; auch in ... vorhergehenden Annäherungen, aufgezeichnet; der zuerst ge-... eichnet.]

$\left[\frac12\left(\frac1{r''}-\frac1r\right)=\right]$ 0,002 1400 125

$[\ldots]$ 7,330 4163

$[\sin\frac12(v''-v)\ldots]$ 8,907 4003

$\left[\frac ep\sin(\frac12(v''+v)-\omega)\ldots\right]$ 8,423 0160

$[\sin(\frac12(v''+v)-\omega)\ldots]$ 9,987 6267

$\left[\frac ep\ldots\right]$ 8,435 3893

$[p\ldots$ 0,]435 5821

$[e=\sin\varphi\ldots]$ 8,870 9714

$[\frac12\sin\varphi\ldots]$ 8,569 9414

$[\cos\frac12\varphi\ldots]$ 9,999 6997 $\quad \varphi = 4^0\,15'\,39''00$

$\log\sin\frac12\varphi =$ 8,570 2417 [*)] $\quad \frac12\varphi = 2\ \ 7\ 49''50$

$\left[\frac12\left(\frac1{r''}+\frac1r\right)-\frac1p\right.$ = 0,00]6 392 5795

$[\ldots]$ 7,805 6761

$[\cos\frac12(v''-v)\ldots]$ 9,998 5777

$\frac ep\cos(\frac12(v''+v)-\omega)\ldots$ 7,807 0984 [n]

$\frac ep\sin(\frac12(v''+v)-\omega)\ldots$ 8,423 0160

$[\text{cotang}(\frac12(v''+v)-\omega)\ldots]$ 9,384 0824 [n]

$[\frac12(v''+v)-\omega=]$ $103^0\,36'\,43''53$

$[\frac12(v''+v)=]$ 73 27 55″20

$[\omega=]$ 329 51 11″67

$[\frac12(v''+v)-\omega=]$ $103^0\,36'\,43''53$

$[\frac12(v''-v)=]$ 4 38 3″96

$[M=v-\omega]$ 98 58 39″57

$[M''=v''-\omega]$ 108 14 47″49

$p\ldots$ 0,435 5821

$b\ldots$ 0,436 7841

$a\ldots$ 0,437 9861

$[\sqrt{a}\ldots]$ 0,218 9930

$[a^{3/2}\ldots]$ 0,656 9791

[mot. diurn. ⊙ med. 3,]550 0071

[mot. diurn. med.] 2,893 0279

[Zur Probe von E und E'']

[S. 3 u. 10] $\quad [\sin M\ldots]$ 9,994 6467 $\quad [\sin M''\ldots]$ 9,977 5947 $\quad [\frac12 M=]$ $49^0\,29'\,19''785$

$[\sin\frac12\varphi\ldots]$ 8,570 2417 $\quad [\sin\frac12\varphi\ldots]$ 8,570 2417 $\quad [\text{tang}(\frac12\varphi+45^0)\ldots]$ 0,032 3264

[Summe] 8,564 8884 $\quad$ [Summe] 8,547 8364 $\quad [\text{tang}\frac12 M\ldots]$ 0,068 3297

$\left[\sqrt{\frac rp}\ldots\right]$ 25031 $\quad \left[\sqrt{\frac{r''}{p}}\ldots\right]$ $-$ 49936 $\quad [\text{tang}\frac12 E\ldots]$ 0,100 6561

$[\sin\frac12(E-M)=]$ 8,562 3853 $\quad [\sin\frac12(E''-M'')]$ 8,542 8428 $\quad [\frac12 M''=]$ $54^0\,7'\,23''745$

$[\frac12(E-M)=2^0\,5'\,31''94$ $\quad [\frac12(E''-M'')=]$ $2^0\,0'\,0''39$ $\quad [\text{tang}(\frac12\varphi+45^0)\ldots]$ 0,032 3264

$[\text{tang}\frac12 M''\ldots]$ 0,140 7051

$[\text{tang}\frac12 E''\ldots]$ 0,173 0315

[*) Anscheinend nach der Formel $\sin\frac12\varphi = \frac12\,\frac{\sin\varphi}{\cos\frac12\varphi}$ gerechnet.]

All the difficulty stems from the great number of variables:

Elements of orbit

w	arg. of perihelion
Ω	long. of ascend. node
i	inclination of orbit
a	semi-major axis
e	eccentricity
l_0	mean heliocent. long.

(A) $\Longleftrightarrow$

Heliocentric coordinates

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

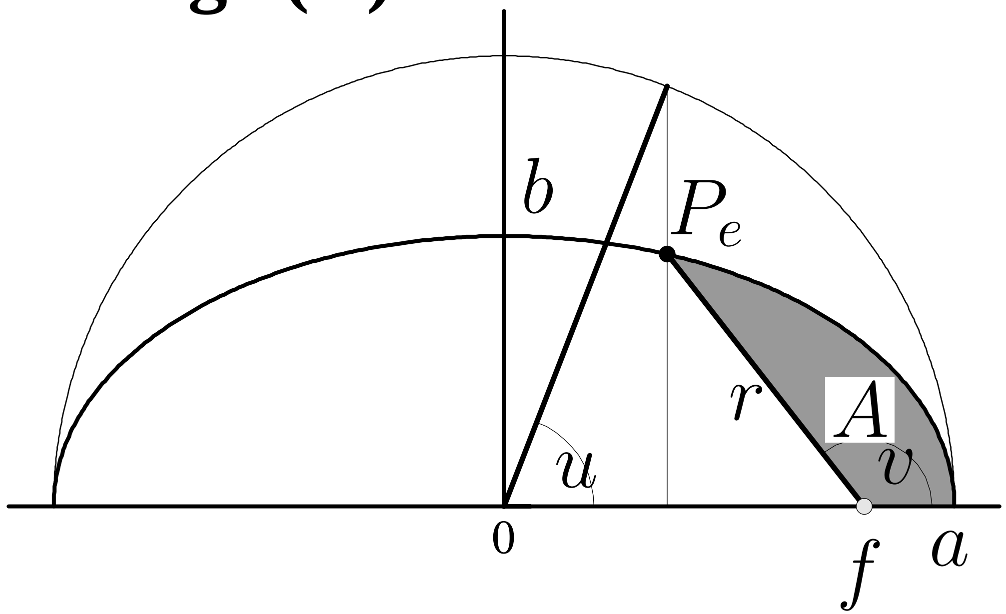
(B) $\Longleftrightarrow$

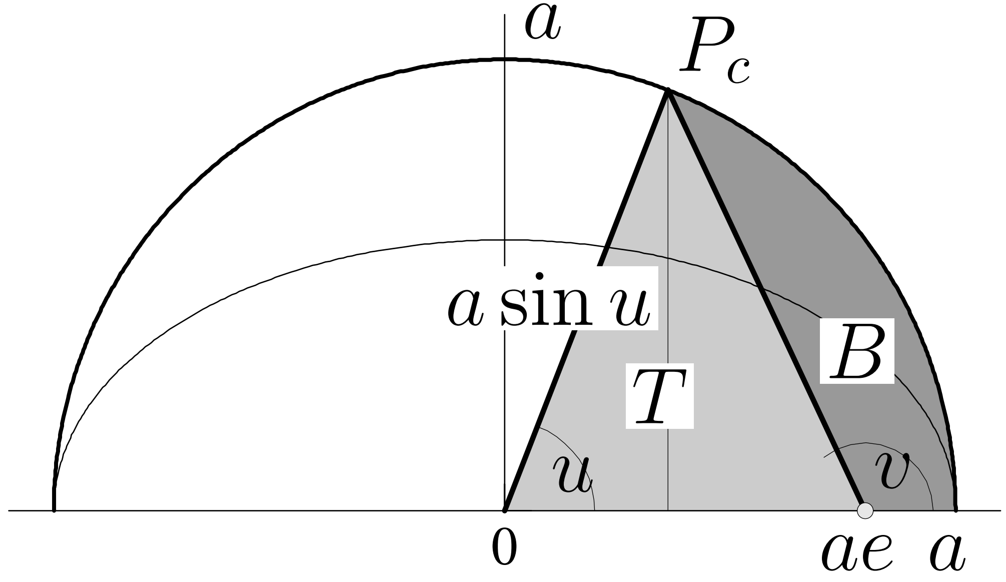
Geocentric spherical coordinates

$$\begin{pmatrix} \rho \\ \lambda \\ \beta \end{pmatrix}$$

The quantities measured are the angles λ and β (the distance ρ is unknown, of course) for several time values, the quantities to be computed are the elements of the orbit. So we need formulas for the connecting passages (A) and (B).

Passage (A).





For given t, must find u.
Kepler's second law ('same times, same areas')

$$\Rightarrow \qquad \frac{A}{ab\pi} = \frac{t}{P}. \quad \Rightarrow \quad nt = u - e \sin u, \qquad n = \frac{2\pi}{P}$$

(Kepler's equation) since $B = \frac{a}{b} A$, $B = \frac{a^2}{2}(u - e \sin u)$.

Kepler's third law: $n^2 a^3$ is a known constant.

$\Rightarrow$ (using spherical trigonometry) coordinates (x, y, z).

Passage (B). For this, we have to know the solar geocentric coordinates (X, Y, Z) (again by Kepler's laws, this time applied to the earth's orbit) and we obtain the geocentric ecliptic coordinates of the planet by adding these and taking spherical coordinates

$$\begin{aligned}
\xi &= x + X &= \rho \cos \beta \cos \lambda \\
\nu &= y + Y &= \rho \cos \beta \sin \lambda \\
\zeta &= z + Z &= \rho \sin \beta .
\end{aligned}$$

Gauss' Procedure. At that time, it was "easy" to solve

$$\begin{pmatrix} \rho_1 \\ \lambda_1 \\ \beta_1 \end{pmatrix}, \begin{pmatrix} \rho_2 \\ \lambda_2 \\ \beta_2 \end{pmatrix} \overset{\text{(B)}}{\Rightarrow} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} \overset{\text{(A)}}{\Rightarrow} \begin{pmatrix} w \\ \Omega \\ i \\ a \\ e \\ l_0 \end{pmatrix}$$

However, ρ_1 and ρ_2 are unknown!

Gauss: very complicated formula manipulations $\Rightarrow$ compute

$$\begin{pmatrix} \lambda_1 \\ \beta_1 \end{pmatrix}, \begin{pmatrix} \lambda_2 \\ \beta_2 \end{pmatrix} \overset{\text{(B)}}{\Rightarrow} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \qquad \begin{pmatrix} \lambda_2 \\ \beta_2 \end{pmatrix}, \begin{pmatrix} \lambda_3 \\ \beta_3 \end{pmatrix} \overset{\text{(B)}}{\Rightarrow} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

Thereby, it was advantageous to have t_2 exactly in the middle.
Gauss started with the data

Jan. 2, Jan. 22, and Feb. 11.

Recomputed repeatedly by changing dates and data.

The Method of Least Squares

> hat man schon Beobachtungen von 1 oder mehrern Jahren . . ., so halte ich den Gebrauch der Differential-Änderung, wobei man eine beliebige Zahl von Beobachtungen zum Grunde legen kann, für das beste Mittel.
>
> (Gauss, *Summarische Übersicht*, publ. 1809)

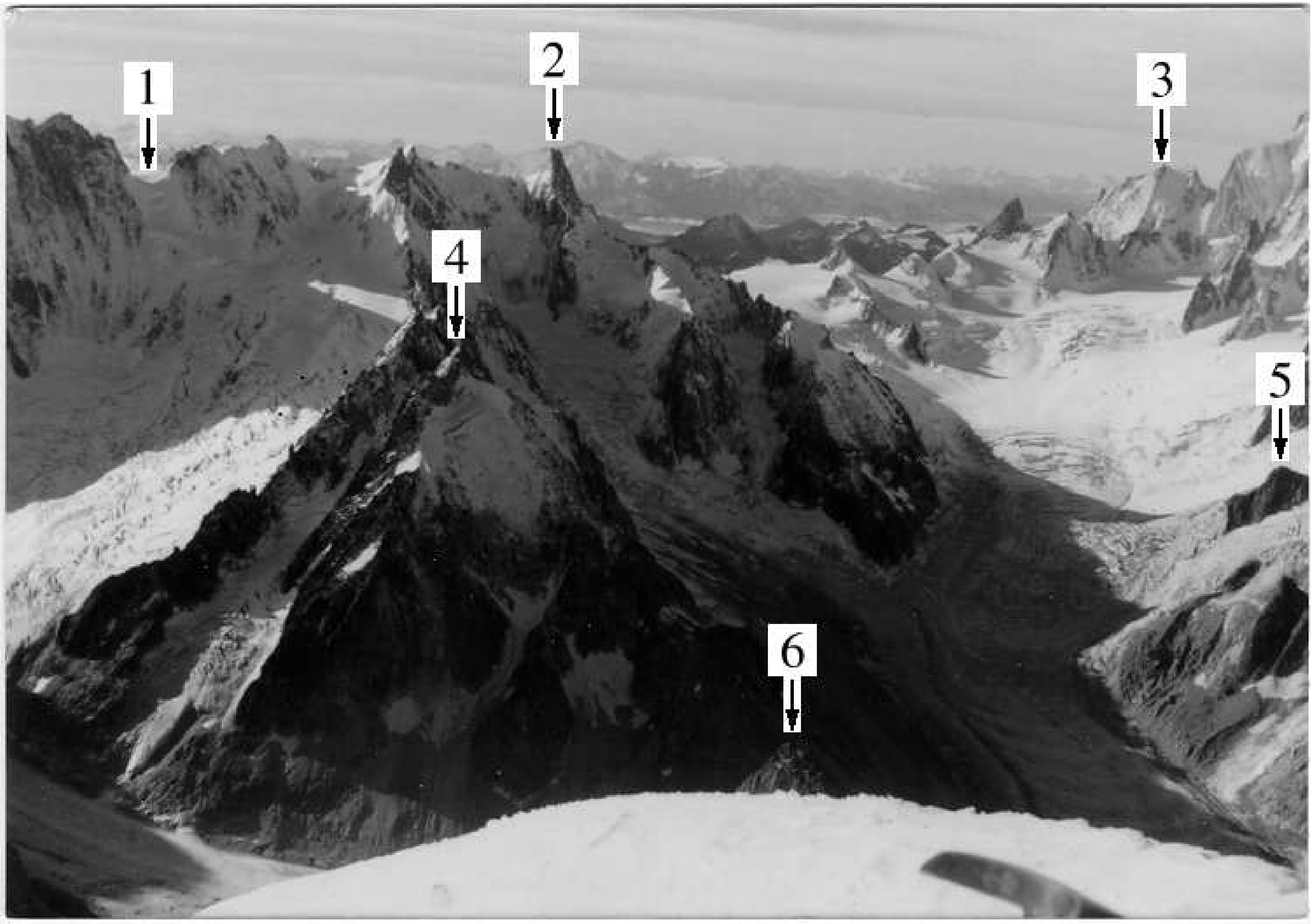
Ceres rediscovered Dec. 1801; $\Rightarrow$ much more data. Start of use of the Method of Least Squares; no publication.

1805: Legendre publishes Nouvelles méthodes pour la determination des orbites des comètes with appendix méthode des moindres quarrés.

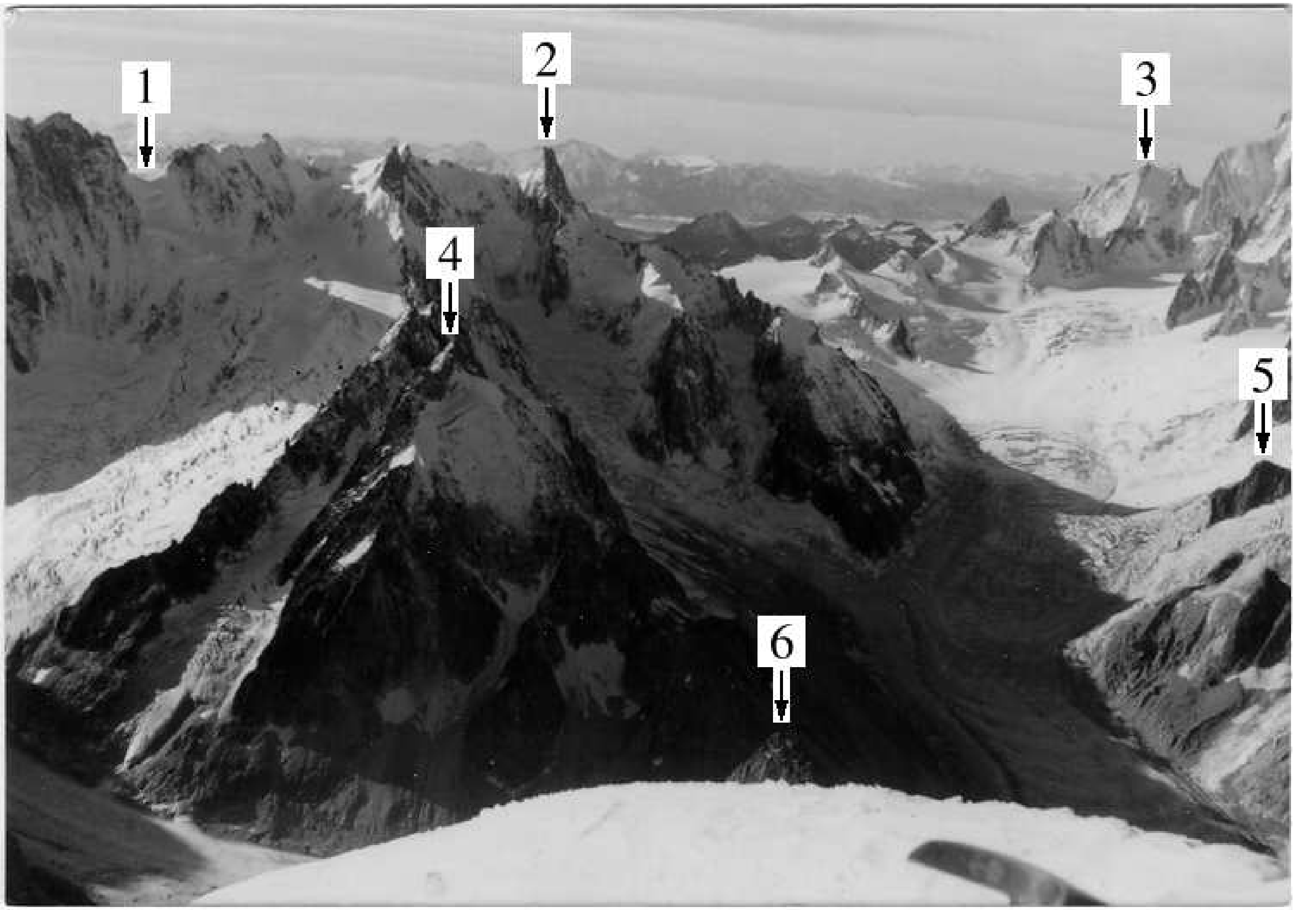
1809: Gauss publishes Theoria motus corporum celestum containing Principium nostrum “which I have made use of since 1795”.

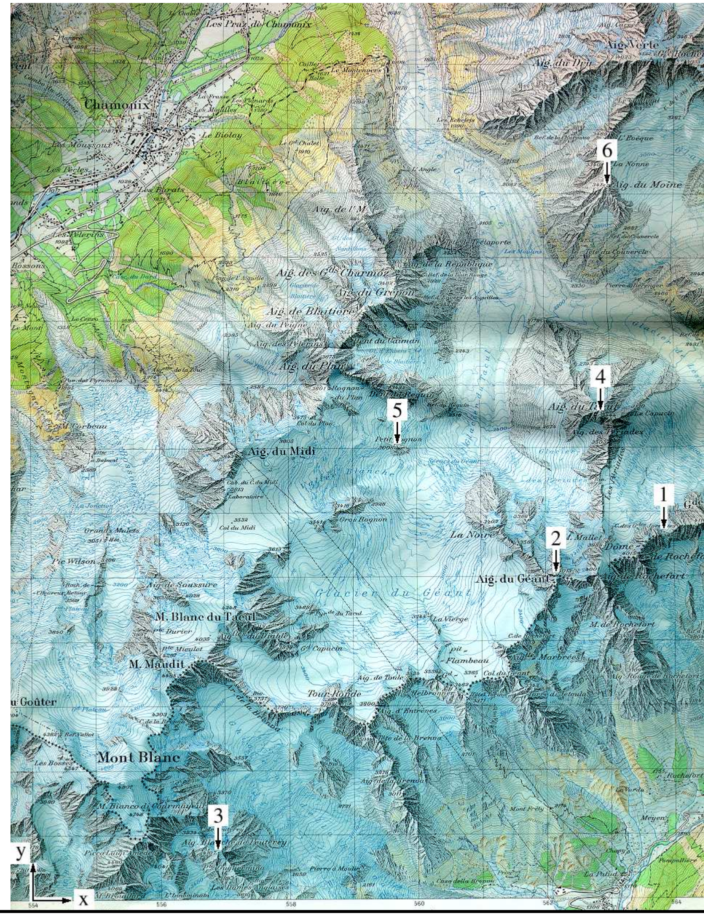
> je n’ai jamais appellé *principium nostrum* un principe qu’un autre avait publié avant moi..
>
> (Legendre in a letter to Gauss, without answer.)

Example: Find the position of the camera !



A photograph (from the Montblanc region)



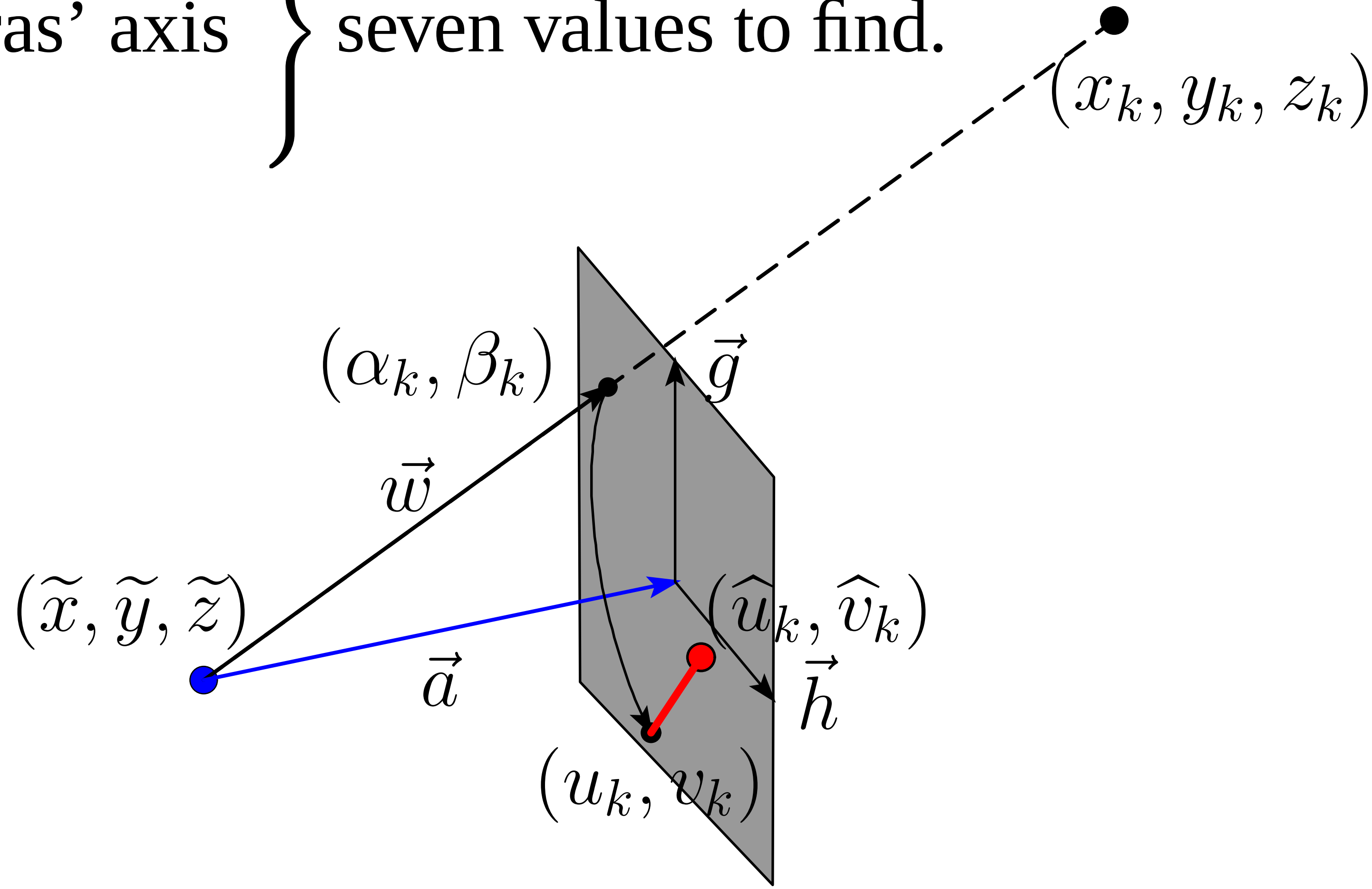


k	$\widehat{u}_k$	$\widehat{v}_k$	x_k	y_k	z_k
1. Col des Grandes Jorasses	−0.0480	0.0290	9855	5680	3825
2. Aiguille du Géant	−0.0100	0.0305	8170	5020	4013
3. Aig. Blanche de Peuterey	0.0490	0.0285	2885	730	4107
4. Aiguille du Tacul	−0.0190	0.0115	8900	7530	3444
5. Petit Rognon	0.0600	−0.0005	5700	7025	3008
6. Aiguille du Moine	0.0125	−0.0270	8980	11120	3412

Solution of our Problem.

$(\widetilde{x}, \widetilde{y}, \widetilde{z})$ objective
$\vec{a} = (a, b, c)$ cameras' axis
θ rotation angle

} seven values to find.



Algorithm:

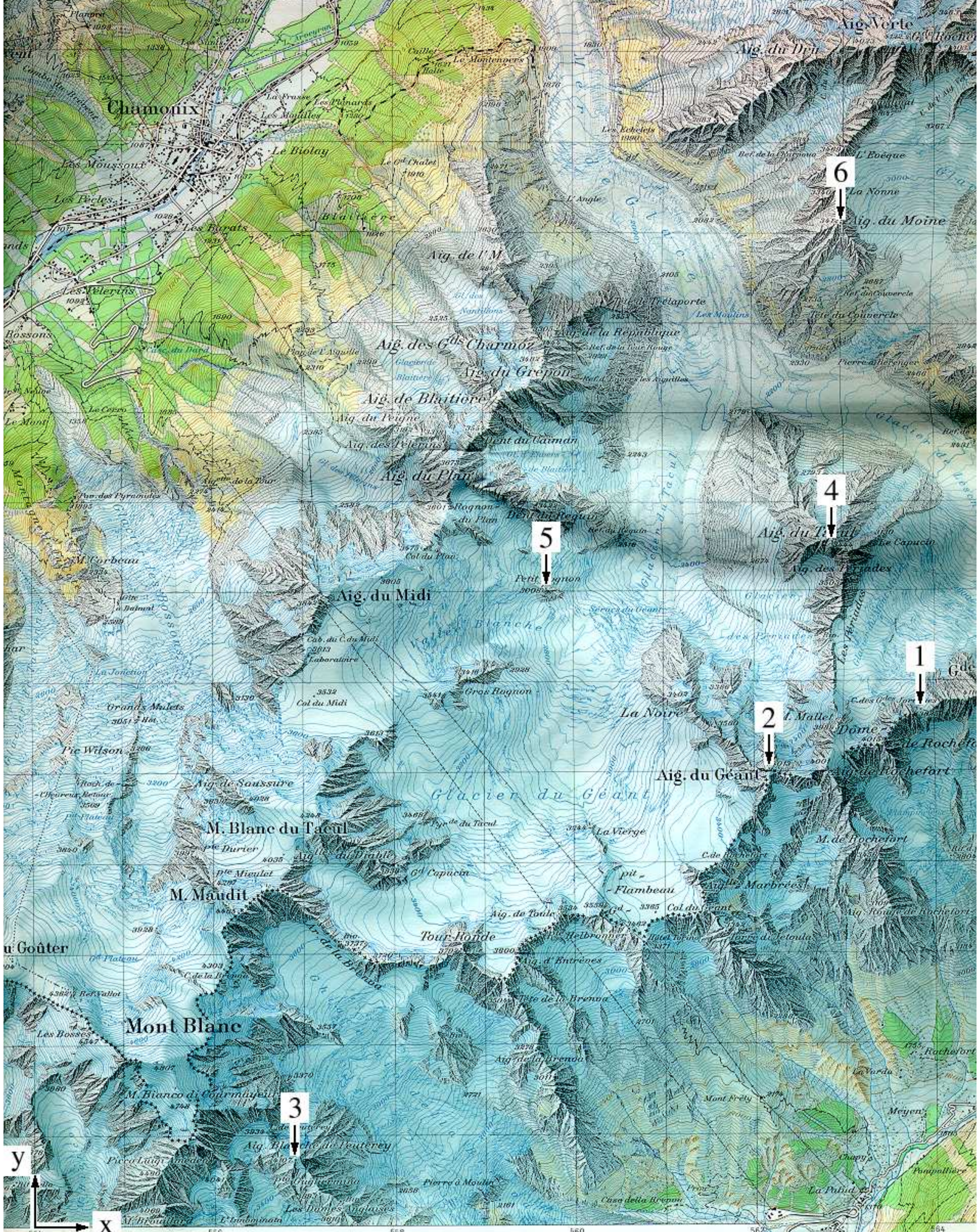
Guess unknowns $\Rightarrow$ compute (u_k, v_k) by el. geometry

minimize $\Omega = \sum_{k=1}^{6} \left((u_k - \widehat{u}_k)^2 + (v_k - \widehat{v}_k)^2 \right)$ by "Differential-Änderung,"

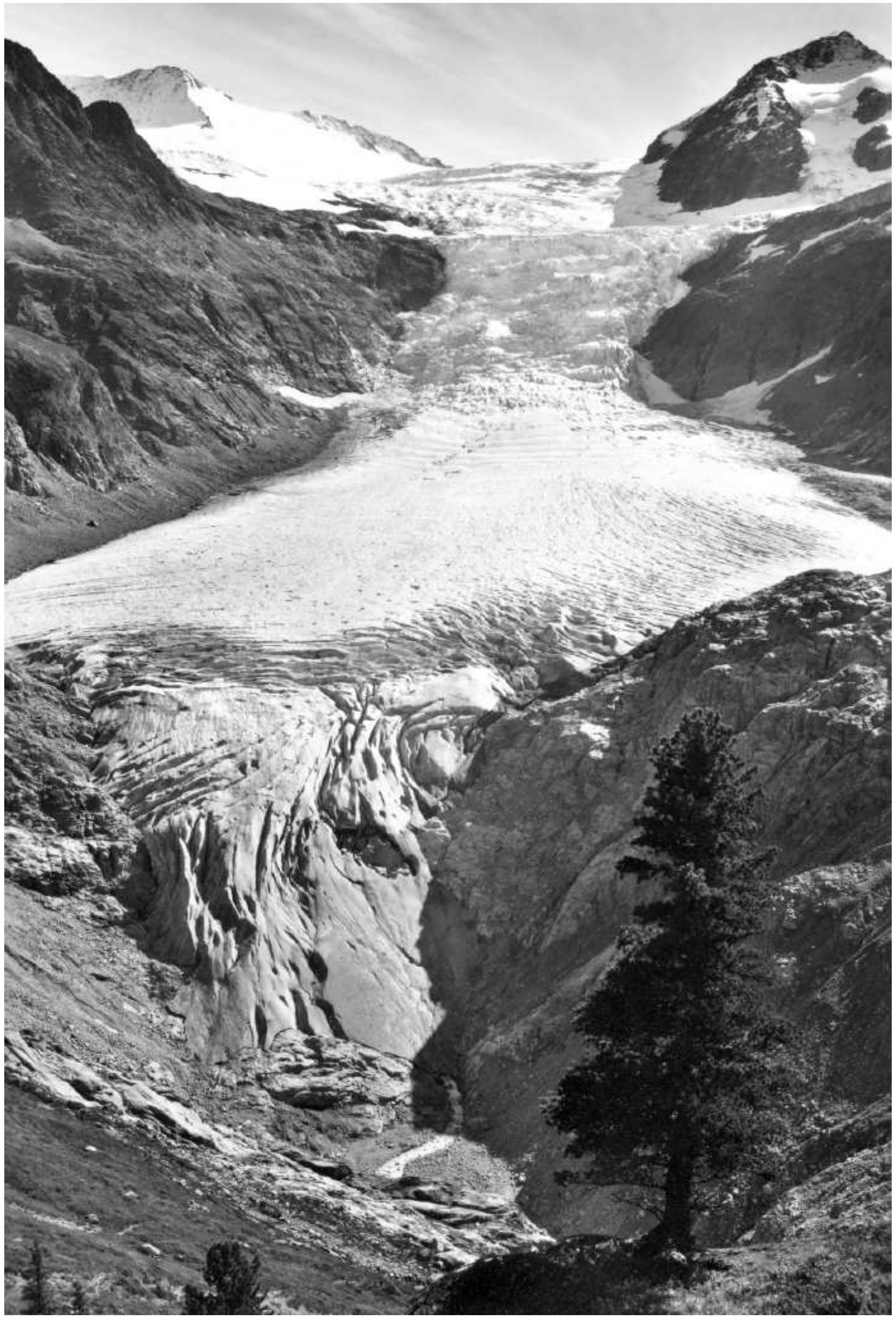
$\Rightarrow$ Solution: $\widetilde{x} = 9679 \quad \widetilde{y} = 13139 \quad \widetilde{z} = 4131.$

$$\widetilde{x} = 9679$$
$$\widetilde{y} = 13139$$
$$\widetilde{z} = 4131.$$

Camera was 8 metres above summit of Aiguille Verte

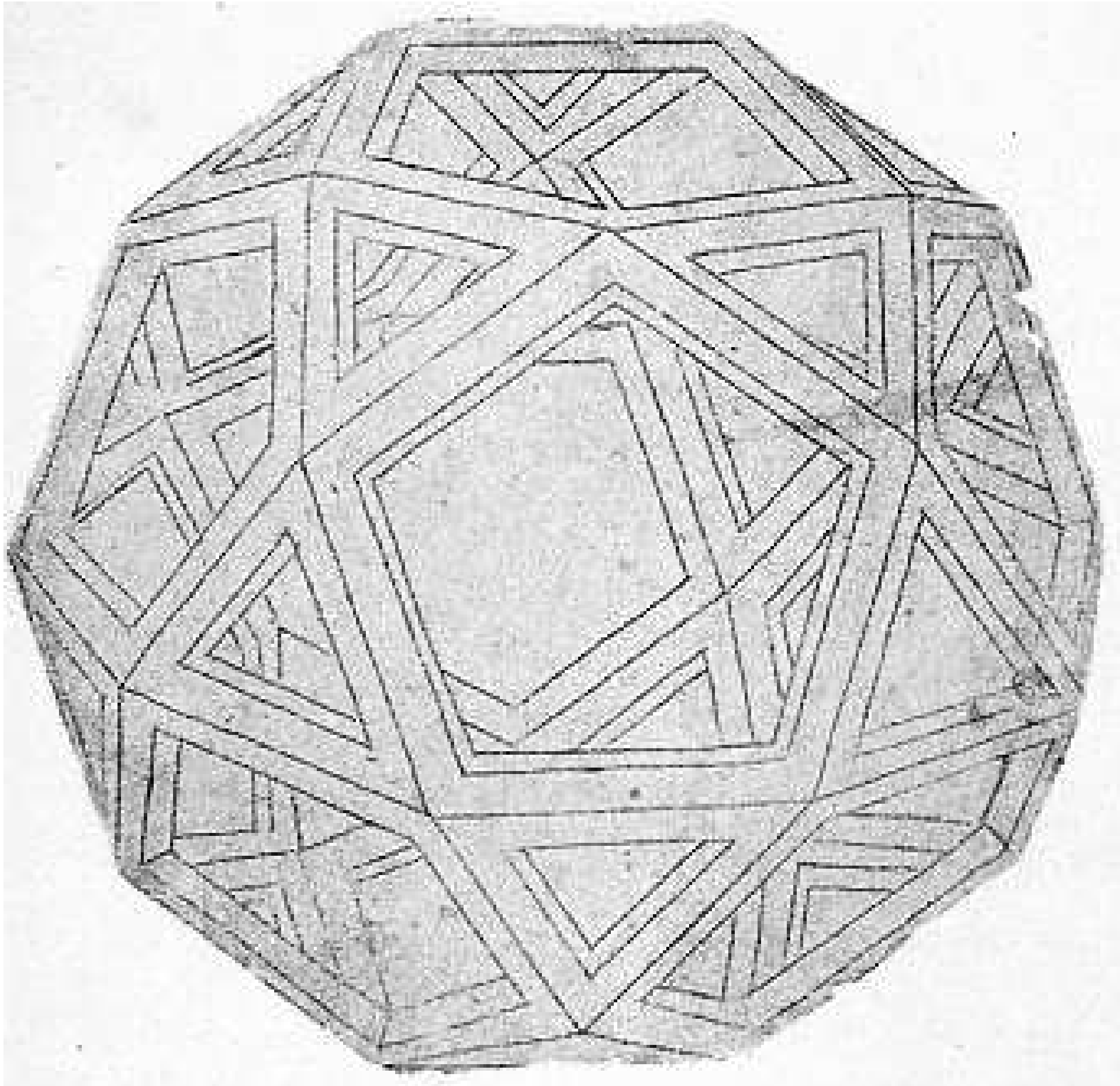


Triftgletscher 1948



Triftgletscher 2004

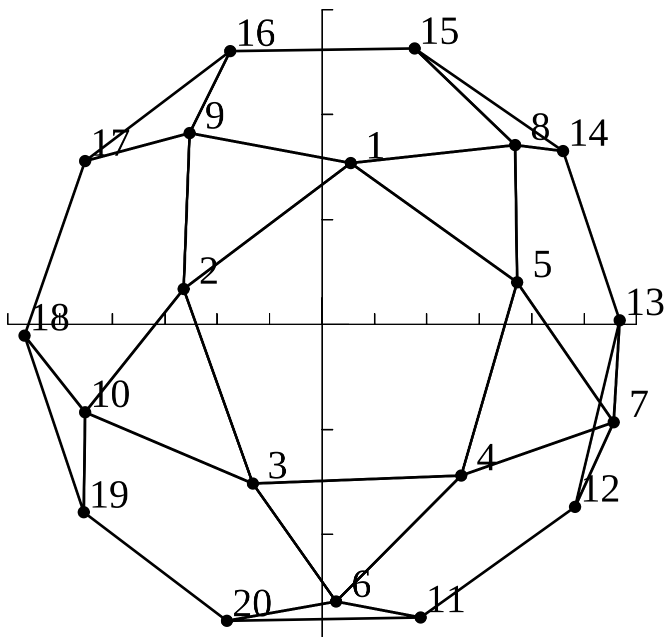
“Correcting” Leonardo da Vinci.

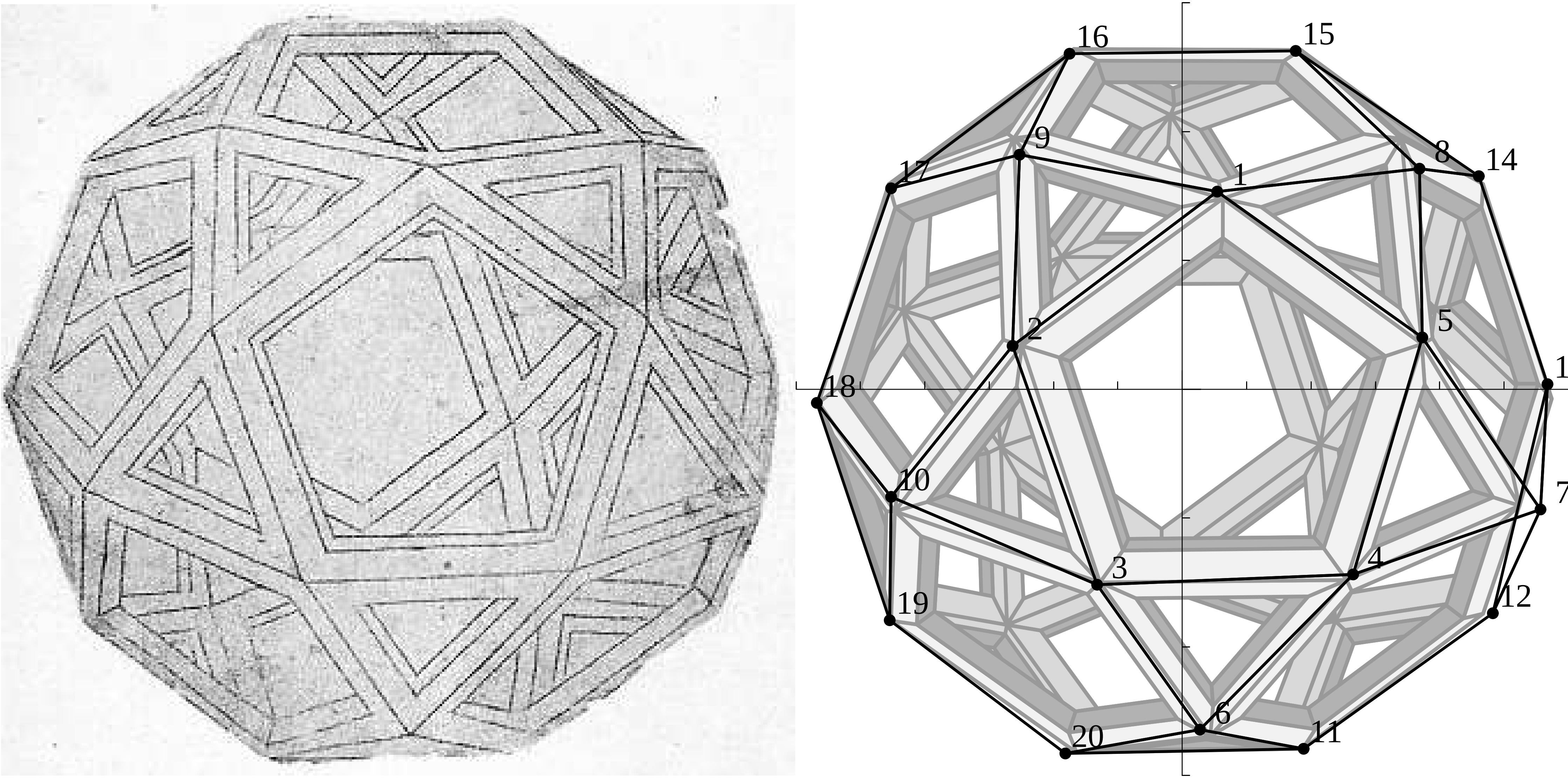


Drawing of Leonardo da Vinci (1510, Codex Atlanticus fol. 707r; Bibliotheca Ambrosiana, Milano)

Is the drawing correct?

k	u_k	v_k
1	5.409	30.691
2	-26.388	6.720
3	-13.259	-30.369
4	26.517	-28.782
5	37.265	8.054
6	2.734	-52.888
7	55.650	-18.639
8	36.865	34.219
9	-25.283	36.394
10	-45.244	-16.728
11	18.814	-55.828
12	48.271	-34.749
13	56.767	0.764
14	46.037	33.043
15	17.609	52.536
16	-17.522	52.122
17	-45.244	31.161
18	-56.768	-2.147
19	-45.433	-35.867
20	-18.198	-56.563



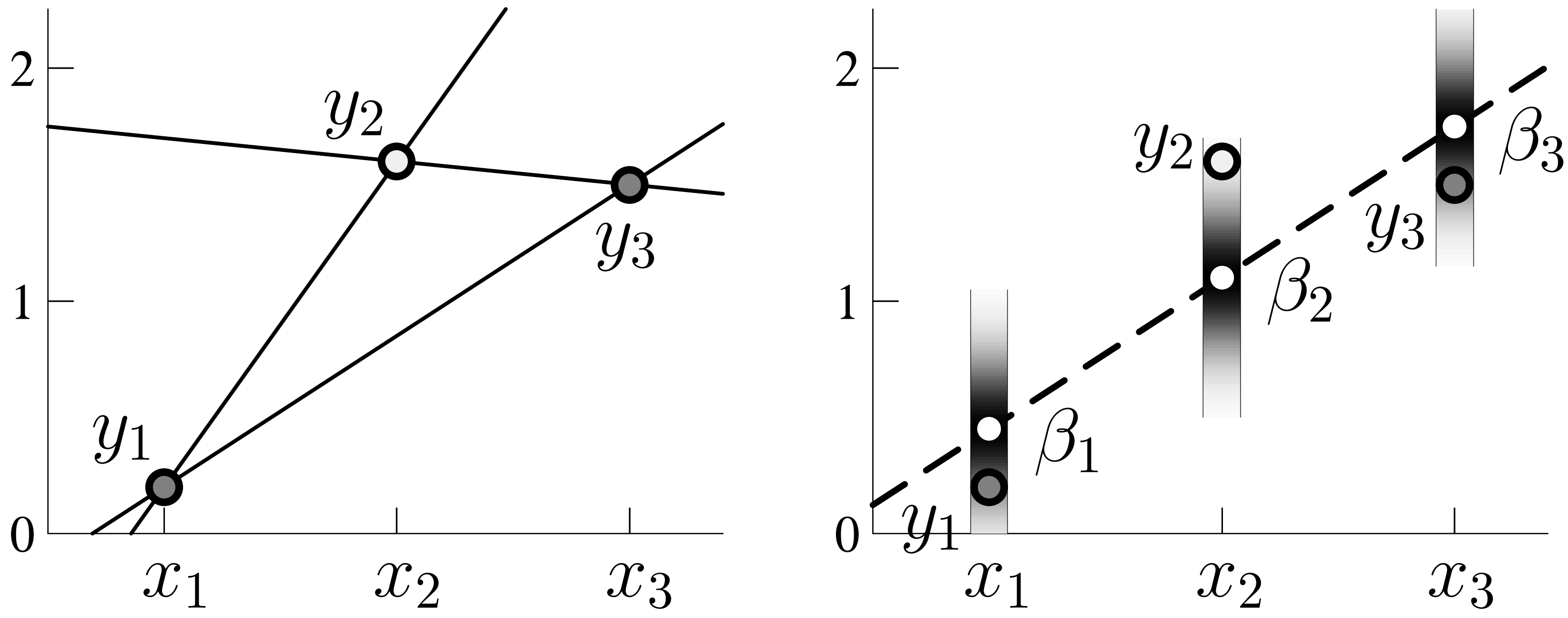


Left: Drawing of Leonardo da Vinci (1510, Codex Atlanticus fol. 707r; Bibliotheca Ambrosiana, Milano); right: Leonardo's vertices and, in grey, the 'corrected' drawing (Assyrus Abdullus & Gerhardus Wannerus, linguæ programmatoriæ Fortranus & Postscriptus, Calculatores SunBlade 100, Universitas Genavæ)

Gauss' Probabilistic Justif. of the Least Squares Principle.

To explain the idea, we treat a simple problem, i.e., the approximation of three 'observations' x_i, y_i ($i = 1, 2, 3$) by an 'orbit' which is a straight line

$$y = a + bx \qquad \Rightarrow \qquad \beta_i = a + bx_i$$



mesures y_i are random samplings.

Probability for having measured y_i (to a precision of Δy):

$$P(0 \leq \beta_i - y_i \leq \Delta y) = \frac{e^{-\frac{(\beta_i - y_i)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \Delta y.$$

Now, the probability for having measures the *three* values y_1, y_2, y_3 (to a precision of Δy) is the product, i.e.,

$$(\frac{\Delta y}{\sigma\sqrt{2\pi}})^3 \prod_{i=1}^{3} e^{-\frac{(\beta_i - y_i)^2}{2\sigma^2}} = (\frac{\Delta y}{\sigma\sqrt{2\pi}})^3 e^{-\frac{\sum_{i=1}^{3}(\beta_i - y_i)^2}{2\sigma^2}}.$$

We have then *maximum likelihood* of our result, when this probability is maximal, i.e., when the exponent

$$\sum_{i=1}^{3} (\beta_i - y_i)^2 = \text{min !} \qquad = \text{principium nostrum!}$$

Grazie !!!