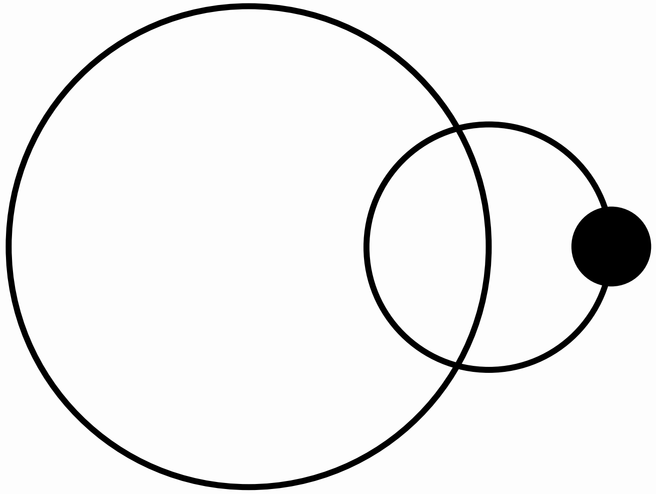
Creating Multidimensional DRAWINGS With Epicycles

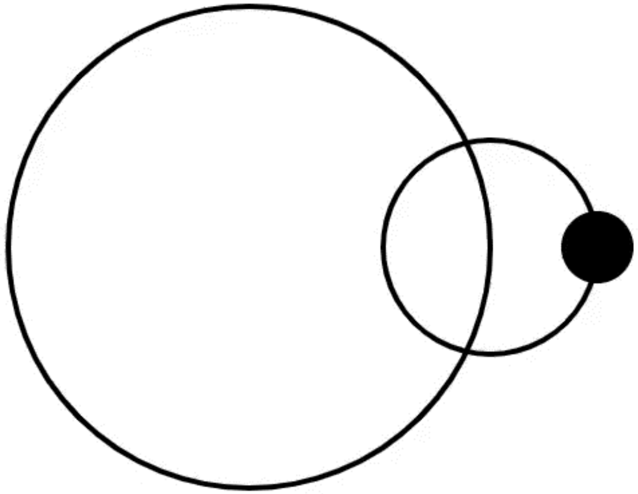
Jan Philipp Birmanns

Supervised by Nicoletta Ravizza-Andri

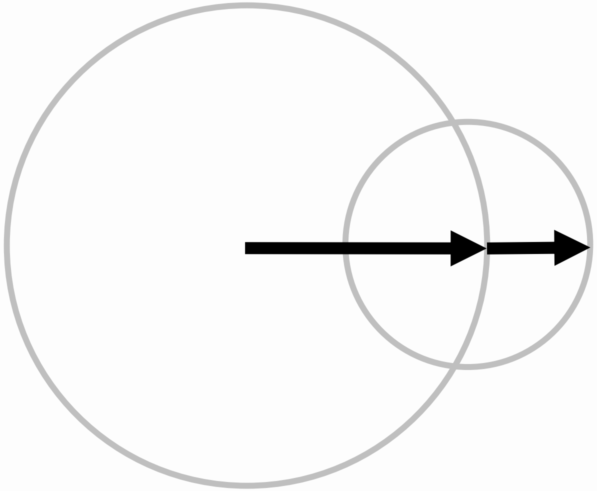
Epicycles



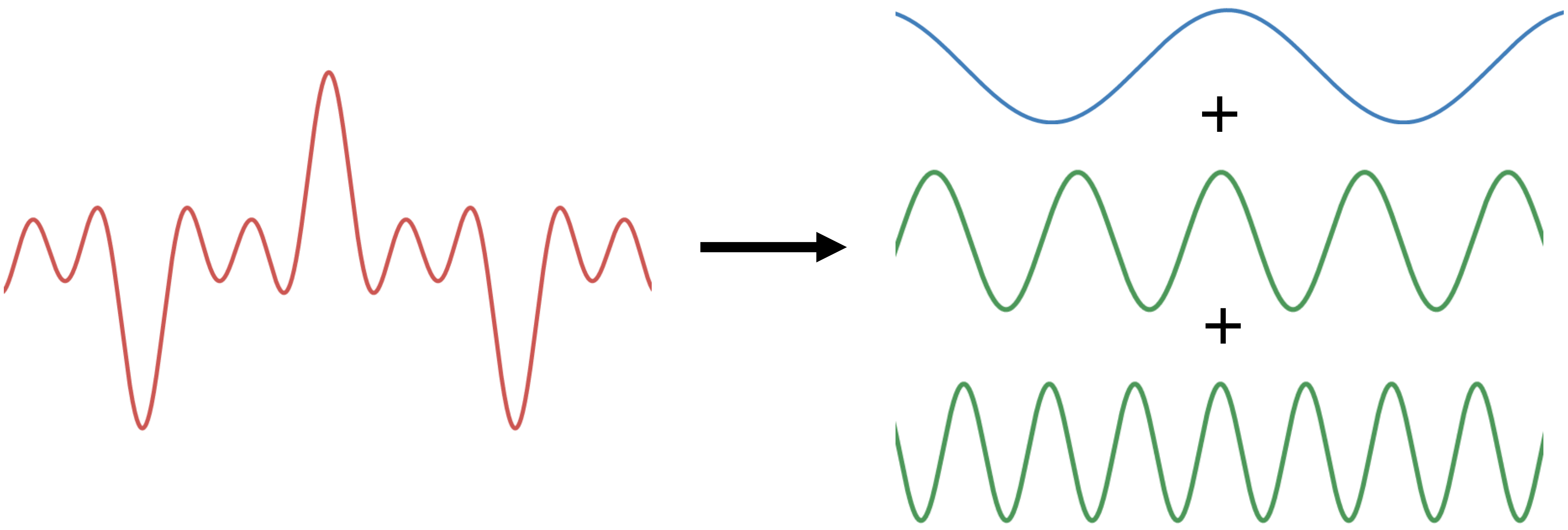
Epicycles



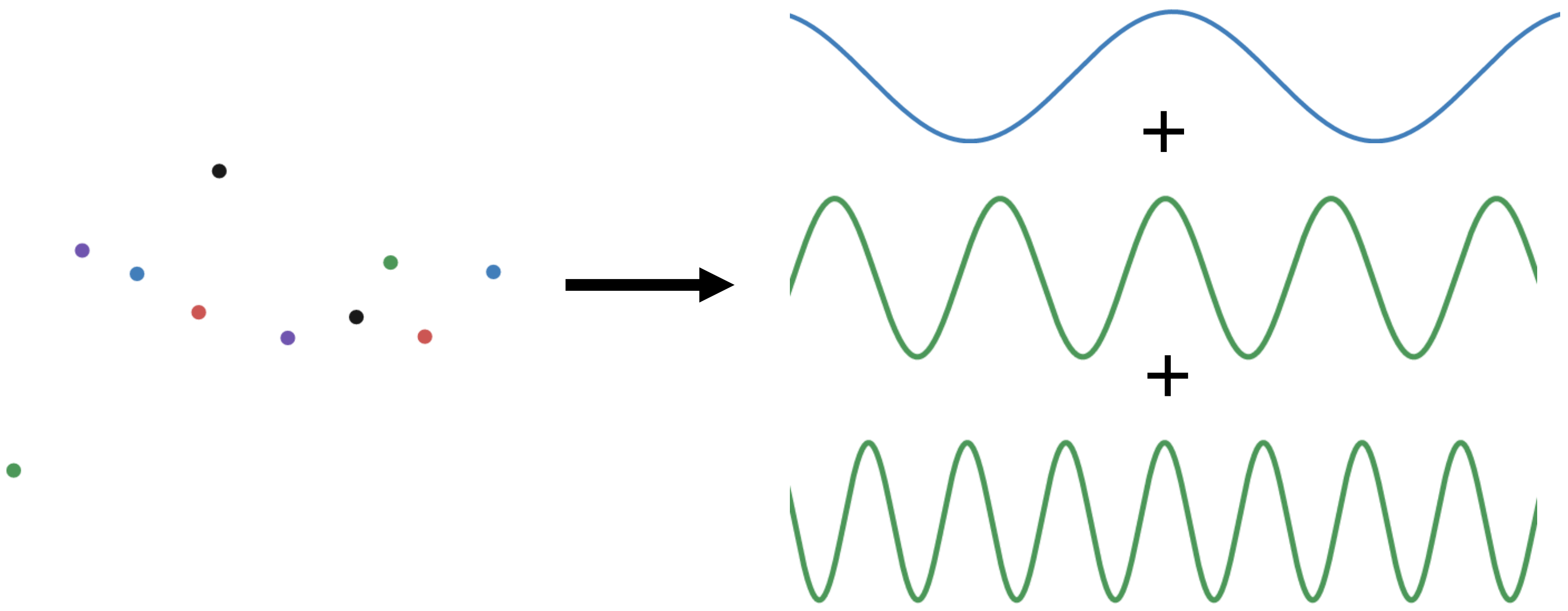
Epicycles



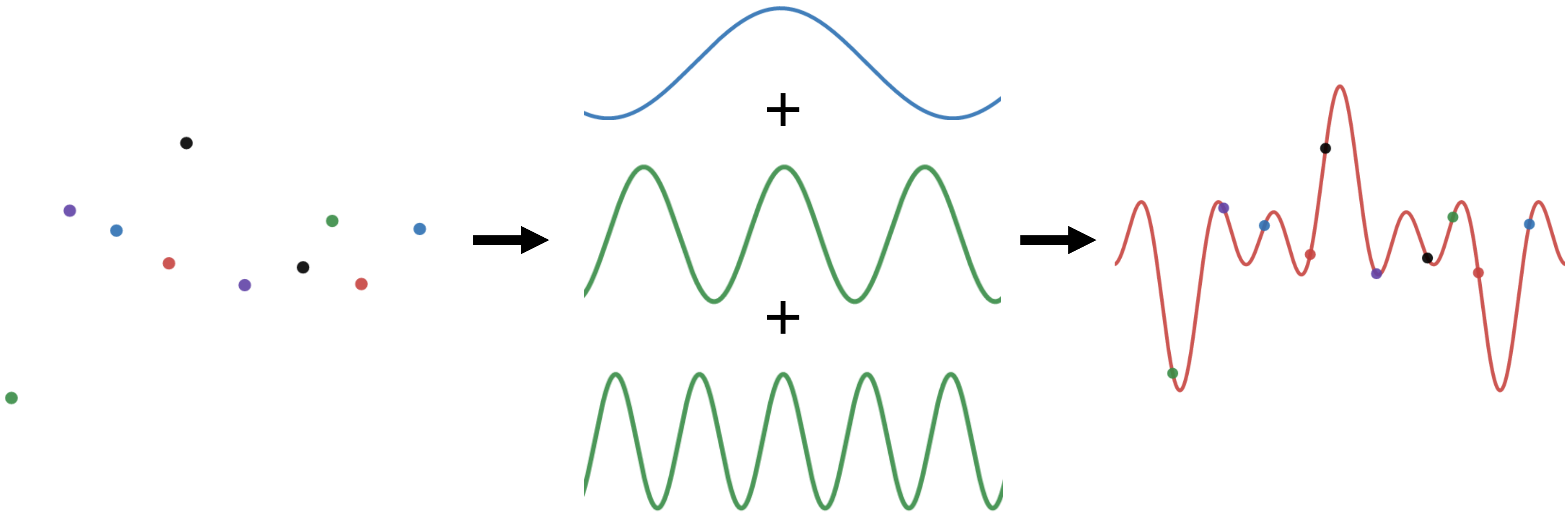
Fourier Transform



Discrete Fourier Transform



Discrete Fourier Transform



Inverse Discrete Fourier Transform

$$x(k) = \frac{1}{N}\sum_{n=0}^{N-1} X_n\, e^{i\cdot 2\pi\cdot n\cdot k\frac{1}{N}}$$

$$x(k) = \frac{1}{N}X_0 e^{i\cdot 2\pi\cdot 0\cdot k\frac{1}{N}} + \frac{1}{N}X_1 e^{i\cdot 2\pi\cdot 1\cdot k\frac{1}{N}} + \frac{1}{N}X_2 e^{i\cdot 2\pi\cdot 2\cdot k\frac{1}{N}} + \cdots$$

Inverse Discrete Fourier Transform

$$\frac{1}{N} X_n e^{i \cdot 2\pi \cdot n \cdot k \frac{1}{N}}$$

$$\frac{1}{N} X_n \left(cos\left(2\pi \cdot n \cdot k \frac{1}{N}\right) + i \cdot sin\left(2\pi \cdot n \cdot k \frac{1}{N}\right)\right)$$

Inverse Discrete Fourier Transform

$$x(k) = \frac{1}{N} X_0 e^{i \cdot 2\pi \cdot 0 \cdot k \frac{1}{N}} + \frac{1}{N} X_1 e^{i \cdot 2\pi \cdot 1 \cdot k \frac{1}{N}} + \frac{1}{N} X_2 e^{i \cdot 2\pi \cdot 2 \cdot k \frac{1}{N}} + \cdots$$

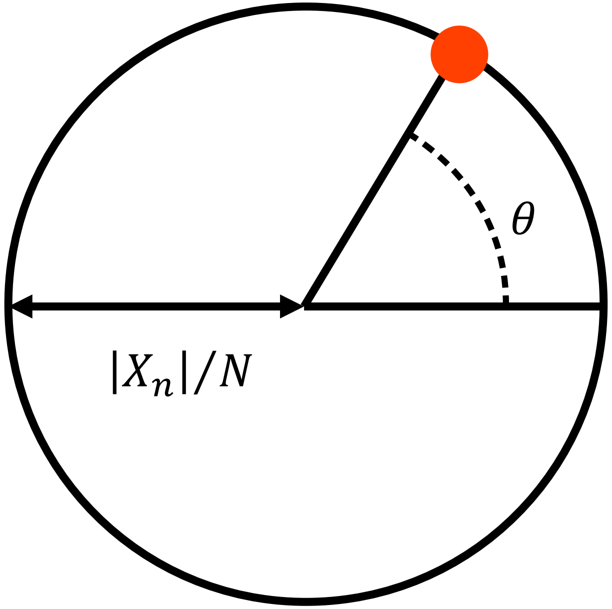
$$\Re x(k) = \frac{1}{N} X_0 cos\left(2\pi \cdot 0 \cdot k \frac{1}{N}\right) + \frac{1}{N} X_1 cos\left(2\pi \cdot 1 \cdot k \frac{1}{N}\right) + \frac{1}{N} X_2 cos\left(2\pi \cdot 2 \cdot k \frac{1}{N}\right) + \cdots$$

Inverse Discrete Fourier Transform

$$\frac{1}{N} X_n \left(cos\left(k \frac{2\pi \cdot n}{N}\right) + i \cdot sin\left(k \frac{2\pi \cdot n}{N}\right)\right)$$

$$P\left(\frac{X_n}{N} cos(\omega), \frac{X_n}{N} sin(\omega)\right)$$

$$\omega = k \frac{2\pi \cdot n}{N} \qquad \theta = tan\left(\frac{\Im X_n}{\Re X_n}\right)$$



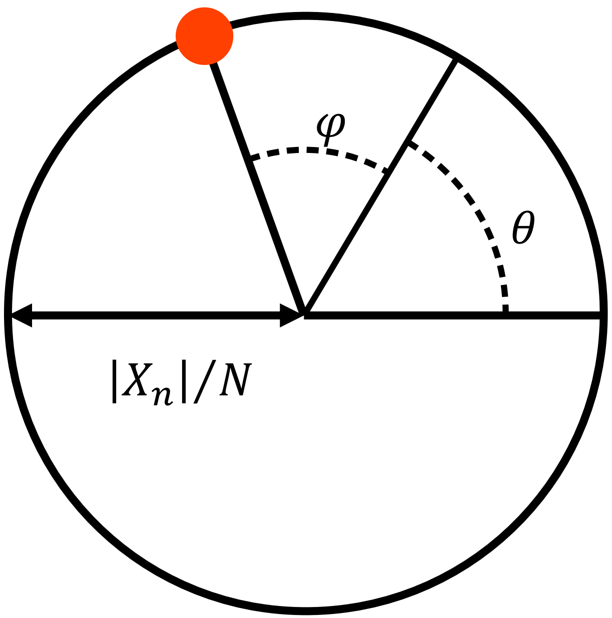
Inverse Discrete Fourier Transform

$$\frac{1}{N} X_n \left(cos\left(k \frac{2\pi \cdot n}{N}\right) + i \cdot sin\left(k \frac{2\pi \cdot n}{N}\right)\right)$$

$$P\left(\frac{X_n}{N} cos(\omega), \frac{X_n}{N} sin(\omega)\right)$$

$$\omega = k \frac{2\pi \cdot n}{N} \qquad \theta = tan\left(\frac{\Im X_n}{\Re X_n}\right)$$

$$\varphi = k \frac{2\pi \cdot n}{N}$$



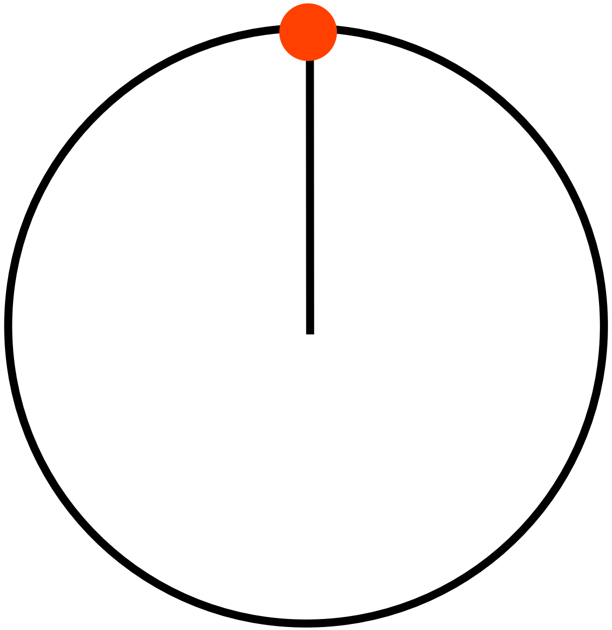
Inverse Discrete Fourier Transform

$$\frac{1}{N}X_n\left(cos\left(k\frac{2\pi\cdot n}{N}\right)+i\cdot sin\left(k\frac{2\pi\cdot n}{N}\right)\right)$$

$$P\left(\frac{X_n}{N}cos(\omega),\frac{X_n}{N}sin(\omega)\right)$$

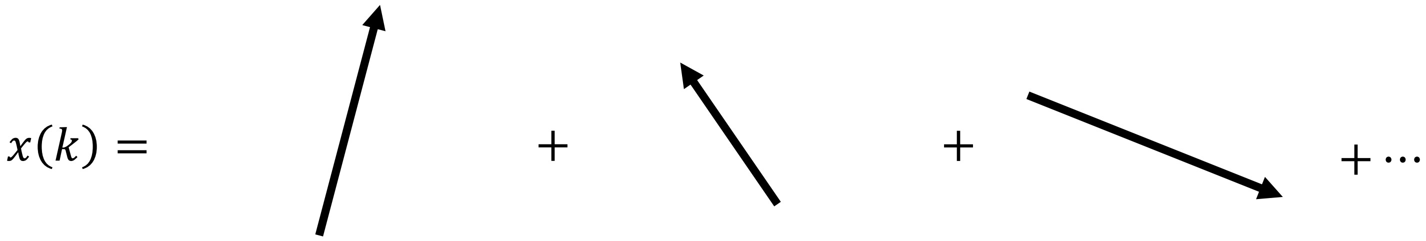
$$\omega = k\frac{2\pi\cdot n}{N} \qquad \theta = tan\left(\frac{\Im X_n}{\Re X_n}\right)$$

$$\varphi = k\frac{2\pi\cdot n}{N}$$



Inverse Discrete Fourier Transform

$$x(k) = \frac{1}{N} X_0 e^{i \cdot 2\pi \cdot 0 \cdot k \frac{1}{N}} + \frac{1}{N} X_1 e^{i \cdot 2\pi \cdot 1 \cdot k \frac{1}{N}} + \frac{1}{N} X_2 e^{i \cdot 2\pi \cdot 2 \cdot k \frac{1}{N}} + \cdots$$



dft.birmanns.org

Expanding to Higher Dimensions

$c = a + bi$ → $q = a + bi + cj + dk$

Discrete Fourier Transform → Discrete Quaternion Fourier Transform

Inverse Discrete Quaternion Fourier Transform

$$x(f) = \sum_{n=0}^{N-1} e^{-\mu 2\pi n f \frac{1}{N}} X_n \qquad \mu = i$$

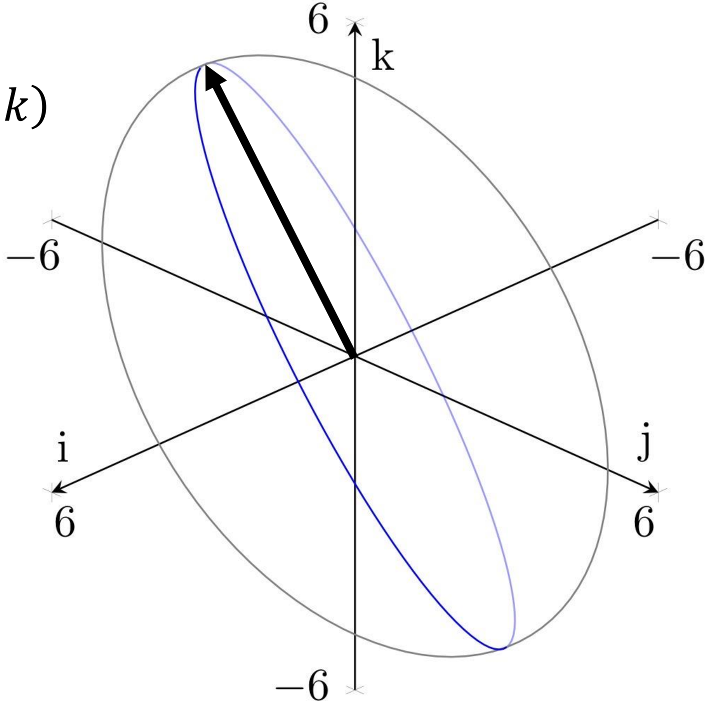
$$e^{-\omega \cdot i} X_n = (cos(\omega)a - sin(\omega)b) + i(cos(\omega)b + sin(\omega)a)$$
$$+j(cos(\omega)c - sin(\omega)d) + k(cos(\omega)d + sin(\omega)c)$$

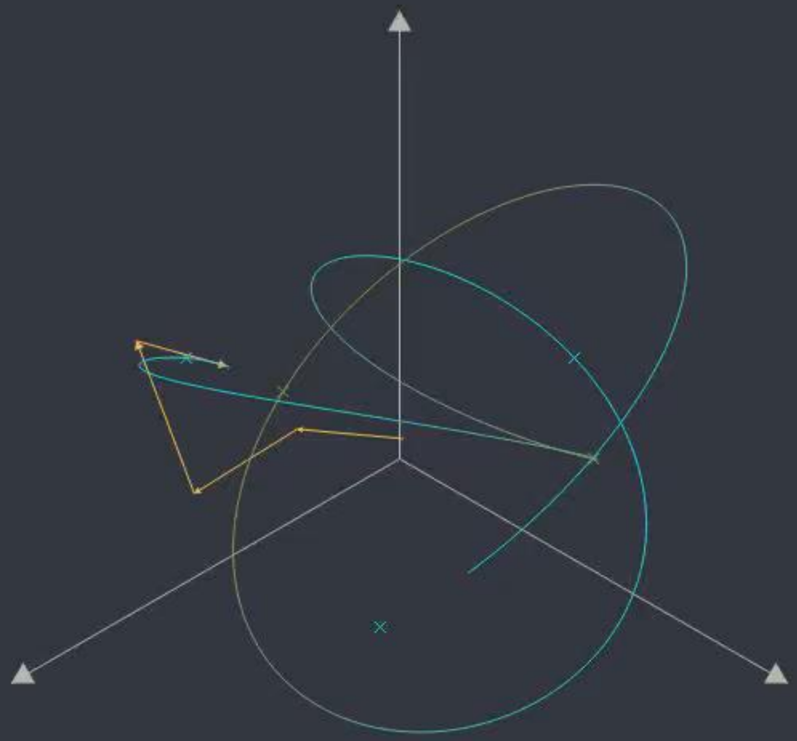
$$X_n = a + bi + cj + dk$$
$$\omega = 2\pi n f \frac{1}{N}$$

$$\left|e^{-\omega \cdot i} X_n\right| = \sqrt[2]{a^2 + b^2 + c^2 + d^2}$$

Inverse Discrete Quaternion Fourier Transform

$(cos(\omega)b + sin(\omega)a)i + (c + di)(cos(\omega)j + sin(\omega)k)$





dqft.birmanns.org